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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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в	г	е	ё	ж	и	й	у	х	ц	ч	ш	щ	ъ	ы	ь	э	ю	я
v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ĭ	′	è	yu	ya

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

November 1950 Vol. 3, No. 11

Communications

Concerning Rev. 3, 1033 of I. Flügge-Lotz and K. Klotter, On the motion of an oscillating system, under the influence of flip-flop controls.

It was not pointed out in the review that the original report appeared in 1943, and hence that it is one of the early papers describing the use of the phase-plane method in analyzing discontinuous control systems. Also omitted was the fact the paper deals with a system whose motion is described by the differential equation $ay + by + cy = NB$, where c, in contrast to most contemporary papers, is finite. The apologies of the reviewer and editor are hereby tendered.

Theoretical and Experimental Methods

(See also Revs. 2179, 2199, 2249, 2465)

2130. **Helmut Epheser, Conformal mapping of simply connected regions which are bounded by arcs of concentric logarithmic spirals** (in German), J. Reine angew. Math. 187, 131–152 (1950).

Mapping $z \to e^z$ transforms a polygon in complex z-plane into a domain D bounded by logarithmic spirals whose asymptotic point is at origin; obviously, D contains neither origin nor point at infinity. Such domains can, therefore, be mapped onto unit circle by means of the Schwarz-Christoffel formula. Author shows that by a slight modification of latter formula it is also possible to treat cases in which D contains origin, point at infinity, or both.
Courtesy Mathematical Reviews Z. Nehari

2131. **W. E. Milne, A note on the numerical integration of differential equations,** J. Research Nat. Bur. Standards 43, 537–542 (1949).

Author offers a numerical method of integrating the ordinary differential equation $y^{(n)}(x)=f(x; y,y', \ldots, y^{(n-1)})$. Method is a step by step integration at equidistant intervals $x_{i+1}-x_i=h$. In case $n=1$, with $y_i^{(m)}=y^{(m)}(ih)$ two consecutive steps are based on the formulas

$$y_1-y_0=\tfrac{1}{2}h(y_1'+y_0')-\tfrac{1}{10}h^2(y_1''-y_0'')+\tfrac{1}{120}h^3(y_1'''+y_0''') \quad (1)$$

$$y_2-2y_1+y_0=7h(y_1'-y_0')-3h^2(y_1''+y_0'')+\tfrac{1}{12}h^3(11y_1'''-5y_0''') \quad (2)$$

which are shown to be equivalent to 6th-order Taylor formulas. Differentiating (1) (for $n=1$) yields

$$y''=f_x+f_y y'; \quad y'''=f_{xx}+2f_{xy}y'+f_{yy}y'^2+f_y y'' \quad (3)$$

Based on (1), (2), and (3) a step by step process is described and illustrated with worked examples. Advantages claimed for method consist of simplification of formulas and elimination of usual high order difference table. On the other hand, because of (3), method is clearly limited to simple, and preferably linear, f. Generalizations to $n>1$ are given.
Courtesy Mathematical Reviews H. O. Hartley, England

2132. **L. Collatz, Method of differences for the numerical integration of ordinary differential equations of *n*th degree** (in German, with Russian summary), Z. angew. Math. Mech. 29, 199–209 (1949).

Author derives formulas for numerical integration of the nth-order differential equation

$$y^{(n)}(x)=f(x; y,y', \ldots, y^{(n-1)}) \quad (1$$

Method is a step by step integration based on a difference table of $y^{(n)}(x)=f$ at equidistant x_r (interval h). Having obtained values of the $y^{(m)}(x)$ for all $x_r \leqq x_r$, $m=0, 1, \ldots, n$, the $y^{(m)}(x_{r+1})$ are computed from

$$y^{(m)}(x_{r+1}) = \sum_{\nu=0}^{n-m-1} (h^\nu/\nu!)y^{(m+\nu)}(x_r)+h^{n-m}\sum_{\rho=0}^{p}\beta_{n-m,\rho}\nabla^\rho f$$

where the $\nabla^\rho f$ are backward differences of the f at $x=x_r$ and the β are tabulated numerical coefficients. Formula is therefore a Taylor expansion at $x=x_r$ in which all differentials up to $y^{(n-1)}$ are utilized (as they are in any case required for the evaluation of the f) and higher differentials are estimated from the difference table of the f. As an alternative to (1) a formula is offered in which the $\nabla^\rho f$ are backward differences at $x=x_{r+1}$ so that iteration is necessary but the tabulated new numerical coefficients β^* converge faster. In its present form the method is suitable when f involves all differentials but modifications are possible if some $y^{(m)}$ are not required. Link with other methods is established, convergence of iteration process investigated, and the "starting off" of solution briefly discussed. Note list of numerous errata.
Courtesy Mathematical Reviews H. O. Hartley, England

2133. William G. Cochran and Gertrude M. Cox, Experimental designs, New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1950, ix + 454 pp. Cloth, 6 × 9 in., $5.75.

Book is an extensive survey of most of the numerous designs that have been developed during last 20 years in field of statistical planning of experiments. It is written mainly for use in experimental research, and it is reviewer's belief it will be highly appreciated as a complement to R. A. Fisher's book *The design of experiments*, and will prove a valuable contribution towards sounder experimentation. Authors have spared no pains in discussing advantages and limitations of various designs, and in giving detailed descriptions of how to solve practical difficulties in statistical analysis. A careful selection of examples, completely worked out, is presented. Most examples are taken from agricultural research, but generality of the methods is in many cases obvious.

First part of book gives a description of statistical methods used in rest of book. It is, however, assumed that the reader already has some basic knowledge of statistics. To each design is then attached a description of appropriate statistical analysis of obtained data or, where special methods are recommended, a refer-

ence to statistical literature. A table of random numbers for use in randomization procedures and a detailed bibliography are given. N. G. Blomqvist, USA

2134. M. Dumas and P. Maheu, Statistical methods and their applications in the field of technology (in French), Mémor. Artill. 23, no. 3, 213 pp. (1949).

Treatise is supposedly an introduction to theory of statistics and its applications in technology. In reviewer's opinion, authors have placed too much emphasis on inverse probability. Reviewer does not see why emphasis is necessary or desirable. While work has a number of interesting industrial examples, it is probably too difficult for a beginner in the field of industrial statistics. Benjamin Epstein, USA

2135. H. C. Hamaker, Lot inspection by sampling, Philips tech. Rev. 11, 176–182 (Dec. 1949).

Expository article. A resumé of known methods for construction of sampling plans for inspection of large lots of material. Benjamin Epstein, USA

2136. A. S. Predvoditelev, A possible classification of statistical systems (in Russian), Vestnik Moskov. Univ. 1947, no. 7, 23–42 (1947).

A criterion for stability of motion of statistical systems is first established. Certain stepwise continuous functions associated with stable statistical systems are introduced. Identities, which are generalizations of those due to Riemann, Hugoniot and Hadamard, are found between above functions. Combination of these identities with Hamilton-Jacobi equation leads to equations of Schrödinger type and of Fokker-Planck-Einstein type, which may be easily transformed one into the other. Transformations used in establishing this connection enable author to define canonical distributions of real, imaginary, and complex moduli. This leads to a natural classification of statistical problems in which classical and quantum statistical systems are shown to represent two aspects of same general principle.
Courtesy Mathematical Reviews G. M. Volkoff

2137. Mauro Picone, New viewpoints in the analysis of periods (in French), Colloques Internationaux du Centre National de la Recherche Scientifique, no. 14, Methodes de calcul dans des problèmes de mecanique, pp. 40–44. Centre National de la Recherche Scientifique, Paris, 1949.

Finding periods for a function $f(t)$ defined in interval $(-\infty, \infty)$ but actually known only in finite interval $(0, T)$ is associated with problem of representing $f(t)$ in $(0, T)$ with a prescribed degree of accuracy by a linear combination of exponential functions $\exp(iy_k t)$, $k=1, 2, \ldots, n$, where $n<N$, the value of N being fixed. This problem in turn is associated with the question whether or not $f(t)$ is solution of a linear homogeneous differential equation with constant coefficients of order $n<N$. Author proposes two methods based on Gramian determinants for finding an answer to this question.
Courtesy Mathematical Reviews W. E. Milne, USA

2138. John Todd, The condition of a certain matrix, Proc. Camb. phil. Soc. 46, 116–118 (1950).

For matrix A having all $a_{ii} = -2$ and all $a_{i,i+1} = a_{i+1,i} = 1$, author finds that the three measures for the ill-conditioning of A: Turing's N-number and M-number, and Wittmeyer-Neumann P-number (given by ratio of greatest and least eigenvalue of A) agree substantially.
Courtesy Mathematical Reviews E. Bodewig, Netherlands

2139. R. C. Prim and C. A. Truesdell, A derivation of Zorawski's criterion for permanent vector-lines, Proc. Amer. math. Soc. 1, 32–34 (1950).

Authors set out purely kinematical questions: (1) What is a ns. (necessary and sufficient) condition that strength of vector tubes of a continuously differentiable vector field $\mathbf{c}$ be same at all cross sections? (2) Given a continuously differentiable velocity field $\mathbf{v}(\mathbf{r},t)$, what is ns. condition that strength of vector tubes of a second continuously differentiable field $\mathbf{c}(\mathbf{r},t)$ remain constant throughout motion? (3) Given a continuously differentiable velocity field $\mathbf{v}(\mathbf{r},t)$, what is a ns. condition that vector tubes of a second continuously differentiable field $\mathbf{c}(\mathbf{r},t)$ be material tubes?

Authors state, with full references to original and alternative derivations, answers to these questions given (1869–1900) by Kelvin, Lamb, and Zorawski, respectively: (1) $\operatorname{div} \mathbf{c}=0$, (2) $D\mathbf{c}/Dt-\mathbf{c}.\operatorname{grad}\mathbf{v}+\mathbf{c}\operatorname{div}\mathbf{v}=0$, (3) $\mathbf{c}\times[D\mathbf{c}/Dt-\mathbf{c}.\operatorname{grad}\mathbf{v}+\mathbf{c}\operatorname{div}\mathbf{v}]=0$. They supply, as main purpose of paper, a simple vectorial derivation of (3). They then remark: "Since (3) is a consequence of (2), but (2) is not generally a consequence of (3), in order for vector tubes of $\mathbf{c}$ to be of strength constant in time, it is necessary, but not sufficient, for vector tubes to be material tubes. Criterion (1) has not been used in deduction of (2) or (3); thus, even if vector tubes are of strength constant in time at each material cross section they are not generally of equal strength at all cross sections."

[To reviewer this conclusion is not clear. The expression "strength of a vector tube" is clearly meaningful if criterion (1) is fulfilled, being (for any t) a number associated with tube. But the "strength at a material cross section" (when not same at all cross sections) needs clarification. Also, it is not clear what is meant by strength of a vector tube being constant in time if the vector tubes are not material tubes; if they are not material tubes, what tubes are compared at different times?]
J. L. Synge, Ireland

2140. C. A. Truesdell and R. C. Prim, Zorawski's kinematic theorems, Nav. Ord. Lab. Mem. 9354, 4 pp. (1947).

Simple vectorial proofs are given of two theorems of Zorawski on kinematics of vector tubes, answering following questions: (1) Given a velocity field $\mathbf{v}(\mathbf{r},t)$, what is a necessary and sufficient condition that vector tubes of a second field $\mathbf{c}(\mathbf{r},t)$ be material tubes? (2) Given a velocity field $\mathbf{v}(\mathbf{r},t)$, what is a necessary and sufficient condition that strength of vector tubes of a second field $\mathbf{c}(\mathbf{r},t)$ remain constant throughout motion?
C. C. Lin, USA

2141. C. A. Swenson, A simple pressure balance, Rev. sci. Instrum. 21, 22–24 (Jan. 1950).

Difficulties of using manometers and Bourdon gages for precise pressure measurements are presented and the practicability of using a pressure balance for measurements is discussed. Author describes a piston-in-cylinder pressure balance built with motor-driven weights to minimize friction. For mechanical stability, piston is set in a re-entrant cavity of weight, thereby providing a very low center of gravity. Performance of author's apparatus is such that measurements are reproducible to plus and minus 0.01 atm at 25 atm. E. Cartotto, USA

2142. Carl G. Hard, Pneumatic measurements of small dilatations (in Swedish), Tekn. Tidskr. 79, 541–546 (1949).

Description is given of deltameter, a pneumatic "foolproof" precision instrument for measuring small displacements, developed by Sandvikens Jernverk A.B. in Sweden. A brief review is given first of original French Solex instrument [see, e.g., Archiv Technisches Messen (July 1947)] and various later de-

velopments. In deltameter, air flows around four measuring disks, rigidly connected by a spindle, such that differential pressure resulting from displacement to be measured is four times that occurring with one disk.

Theory of instrument is given and many advantages are claimed, among them a linear response over a large part of range, possibility of incorporating selsyn or other devices for remote indication, and large magnification ratio (which may reach 10^6). Construction of instrument as a precision comparator and of its constant-pressure-regulator are described, and applicability of the principle for various other purposes is mentioned.

F. J. Plantema, Holland

2143. John W. Miles, A note on Riemann's method applied to the diffusion equation, Quart. appl. Math. 8, 95–101 (1950).

The equations considered are of the form

$$L(u) = \frac{\partial}{\partial x}\left[x^{m-1}\frac{\partial}{\partial x}u(x,t)\right] - kx^{m-1}\frac{\partial}{\partial t}u(x,t) = -Q(x,t) \quad [1]$$

They correspond for positive integers m to heat conduction with spherical symmetry in an m-dimensional medium. A mixed Dirichlet-Neumann problem for u is to be solved. Let R be a region in the (ξ, τ)-plane bounded by a segment C_1 given by $\tau = \text{constant} = t$, $\alpha \leqq \xi \leqq \beta$, and a curve C_2, for which $\tau < t$. A solution u of (1) is to be found on C_1 from the condition that

$$u(\xi,\tau) + \beta(\xi,\tau)\frac{\partial}{\partial \xi}u(\xi,\tau) = \xi^{m-1}f(\xi,\tau) \quad [2]$$

on C_2. Using Green's identity the determination of u is reduced to that of a Riemann function $v(x,t,\xi,\tau)$, which satisfies the homogeneous differential equation adjoint to (1) in R, satisfies a homogeneous condition similar to (2) on C_2, and the condition

$$k\xi^{m-1}v(x,\ t,\ \xi,\ t) = \delta(x-\xi)$$

where δ is the Dirac function.

Question of existence of such a function v and verification of the boundary condition for u are left open. It is pointed out, however, that a necessary condition for existence of v is that $\beta = 0$ wherever $\cos(n, \xi) = 0$ on C_2. Some attention is given to a free-moving boundary in the medium, which occurs in connection with freezing and melting of solids.

Courtesy Mathematical Reviews F. John, USA

2144. B. V. Bulgakov and N. T. Kuzovkov, On the accumulation of disturbances in linear systems with varying parameters (in Russian), Prikl. Mat. Mekh. 14, 7–12 (1950).

Consider the vector-matrix system $dz/dt = F(t)z + w(t)$, $z(0) = c$, a constant vector, where z and w are column vectors and $F(t)$ is a matrix. The problem is that of determining a bound on z in terms of the bounds on $w(t)$ and the solution of the homogeneous equation $dy/dt = F(t)y$. Let $Y(t)$ denote the solution of $dY/dt = F(t)Y$, $Y(0) = I$. Then

$$z = y + \int_0^t Y(t)Y^{-1}(t_1)w(t_1)dt_1$$

whence

$$||z|| \leqq ||y|| + \int_0^t ||Y(t)Y^{-1}(t_1)||\ ||w(t_1)||dt_1,$$

where we set $||y|| = \Sigma_i|y_i|$, $||A|| = \Sigma_{i,j}|a_{ij}|$, if y has the components y_i, A the elements a_{ij}. Difficulty lies in estimating $||Y(t)Y^{-1}(t_1)||$. This can be done, with a certain amount of numerical calculation, if $F(t)$ is a constant matrix, since $Y(t)Y^{-1}(t_1) = Y(t-t_1)$ in this case. Consequently, authors divide t-range into intervals $[0, t_1]$, $[t_1, t_2]$, and so on, and take $F(t)$ constant and equal to $F(t_i)$ in ith interval. Numerical examples are given.

R. Bellman, USA

2145. A. Myškis, On the solutions of linear homogeneous differential equations of the first order of unstable type with a retarded argument (in Russian), Doklady Akad. Nauk SSSR 70, 953–956 (1950).

Under consideration is the equation

$$y'(x) - M(x)y(x - \Delta(x)) = 0 \quad (1)$$

($M(x) \geqq 0$, $\Delta(x) \geqq 0$; $A \leqq x < B$; finite A, B). Let E_0 consist of A and of the set of values $x - \Delta(x) \leqq A$. On E_0 there is assigned a function $\varphi(x)$; $M(x)$, $\Delta(x)$, $\varphi(x)$ are continuous; $y(x)$ is to satisfy (1) so that $y(x - \Delta(x)) \equiv \varphi(x - \Delta(x))$, wherever $x - \Delta(x) < A$, and $y(A) = \varphi(A)$; $\delta_0 = \inf \Delta(x)$, $\Delta_0 = \sup \Delta(x)$, $m_0 = \inf M(x)$, $M_0 = \sup M(x)$, on $[A, B]$; $\varphi_0 = \inf \varphi(x)$, $\Phi_0 = \sup \varphi(x)$, on E_0; with a in $[A, B]$, let $\gamma(a)$ be the upper bound of those x in (a, B), for which $x - \Delta(x) < a$. It has been proved previously [Myškis, Uspehi Matem. Nauk (N.S.) 4, no. 5 (33), 99–141 (1949)] that there is a unique solution on $[A, B]$. Some of the results are as follows. If $\varphi_0 \geqq 0$, $y(x)$ is monotone nondecreasing; if $\varphi_0 > 0$ and $m_0 > 0$, then $y(x)$ is monotone increasing. If $A \leqq a = \gamma(a) < B$, then: (a) if $y(a) = 0$, $y(x) \equiv 0$ for $x \geqq a$; (b) if $y(a) > 0$, $y(x)$ is monotone nondecreasing for $x \geqq a$; (c) if $y(a) > 0$ and $m_0 > 0$, $y(x)$ is monotone increasing. Let $A \leqq a < \gamma(a) < B$; (a) if $y(x) \geqq 0$ on $[a, \gamma(a)]$, $y(x)$ is monotone nondecreasing for $x \geqq \gamma(a)$; (b) if $y(x) > 0$ on $[a, \gamma(a)]$ and $m_0 > 0$, y is monotone increasing for $x \geqq \gamma(a)$.

Given equation (2) $y'(x) - M_1(x)y(x - \Delta_1(x)) = 0$ ($A \leqq x < B$), with the initial function $\varphi_1(x)$ and solution $y_1(x)$; (a) if $0 \leqq M(x) \leqq M_1(x)$, $0 \leqq \varphi(x) \leqq \varphi_1(x)$, $\Delta(x) = \Delta_1(x)$, then $y_1(x) \geqq y(x)$; (b) if $0 \leqq M(x) \leqq M_1(x)$, $\Delta_1(x) \leqq \Delta(x)$ and $\varphi(x)$ ($\varphi_1(x)$) is given also on $E_{1,0}$ (E_0), while therein $0 \leqq \varphi(x) \leqq \varphi_1(x)$, and $\varphi(x)$ ($\varphi_1(x)$) is monotone nondecreasing on $E_0 + E_{1,0}$, then $y_1(x) \geqq y(x)$. Write $a_0 = A$, $a_{k+1} = \gamma(a_k)$ ($k = 0, 1, \ldots$); let T be the upper bound of the right endpoints of those $[a_k, a_{k+1}]$ on which $y(x)$ has zeros. Suppose $T < B$; (a) if $T = \lim a_k$, then $y(x) \equiv 0$ ($x \geqq T$); (b) if $T = a_k$, $a_{k+1} < B$, then $|y(x)|$ is monotone increasing ($x \geqq a_{k+1}$). Moreover, $|y(x)| \leqq \sup_{E_0} |\varphi(x)| \cdot (M_0\Delta_0)^\nu$ ($A \leqq x < T$), where $\nu = \Delta_0^{-1}(x - A)$.

Courtesy Mathematical Reviews W. J. Trjitzinsky, USA

2146. B. F. Bilov, On an estimate for the characteristic numbers of the solution of almost diagonal systems of linear differential equations (in Russian), Prikl. Mat. Mekh. 14, 114–116 (1950).

Paper deals with a regular system $x_i = \Sigma\, p_{ij}(t)x_j$, ($i, j = 1, \ldots, n$) with coefficients continuous and bounded for $t \geqq 0$ and such that $p_{ii} \geqq p_{i+1,\,i+1} + 2(n-1)Q$, where $Q = \max_{ij} |p_{ij}|$ at $j \neq i$. For such system there are given complicated bounds for the characteristic numbers in the sense of Liapounoff.

S. Lefschetz, USA

2147. John Todd, The condition of certain matrices. I, Quart. J. Mech. appl. Math. 2, 469–472 (1949).

Author considers problem of solving the equation $Z_{xx} + Z_{yy} = \lambda Z$ by difference techniques. He replaces this differential equation by the difference equations

$$Z_{r-1,s} + Z_{r,s-1} - 4Z_{r,s} + Z_{r-1,s} + Z_{r,s+1} = h^2\lambda Z_{r,s},$$

where $Z_{r,s} = Z(rh, sh)$; here h is length of side of each elementary square in mesh. Paper is devoted to a discussion of the "condition" of matrix of coefficients of the difference equations; this is a measure of the precision with which matrix can be in-

verted by numerical means. He uses two measures of "condition": (1) the so-called P-condition number, which is the ratio of magnitudes of the largest to the smallest proper values of a symmetric matrix, and (2) the so-called N-condition number, which is the norm of matrix times norm of its inverse divided by order ν of matrix. It is shown that matrix of the difference system has P-condition number $O(\nu)$ and N-condition number $O(1)$. Corresponding results for certain second-order total difference systems are $O(\nu^2)$ and $O(\nu^{1/2})$. Thus, solution of the difference system with two independent variables should be more stable than corresponding system with one variable. In closing, author remarks that, as number m of independent variables increases, the condition of matrix improves and that this last result will be subject of a future paper.

Courtesy Mathematical Reviews H. H. Goldstine, USA

2148. J. C. Jaeger, An introduction to the Laplace transformation with engineering applications, London: Methuen & Co., Ltd.; New York: John Wiley & Sons, Inc., 1949, viii + 132 pp., $1.50.

Monograph presents a substantial treatment of the portion of applications and formal theory of Laplace transformation that is independent of theory of functions of complex variables. It contains an interesting variety of problems in electric-circuit theory, including vacuum tubes, servomechanisms, mechanical analogs to circuit problems, practical methods of determining conditions for stable solutions of circuit problems, and a number of problems concerning transmission lines. Book is divided into four chapters: Fundamental theory, Electric circuit theory, Further theorems and their applications, and Partial differential equations. Book should well serve author's objective of furnishing a good working knowledge of operational methods to students whose mathematical equipment is limited; it should also serve as a valuable reference to those interested in extending their knowledge of applications. A few exercises are presented. R. V. Churchill, USA

2149. Harold T. Davis, A bibliography and index of mathematical tables (with the assistance of Vera Fisher), Evanston, Ill., 1949, xxii + 286 pp. Mimeographed, cloth, 7 × 9 in., $4.

Bibliography consists of 196 pages of titles. There is an index referring from the subject of a table to the pertinent bibliographic entries, and an index referring from the author to the subject of the table. Introduction contains a historical survey of tabulating activities. Ed.

2150. C. J. Bouwkamp, On the evaluation of certain integrals occurring in the theory of the freely vibrating circular disk and related problems, Proc. Kon. Ned. Akad. Wet. 52, no. 9, 987–994 (1949).

Author considers the repeated integral

$$I_{n,m}(\rho) = {}_0\int^1 P_{2n+1}(1-\rho'^2)^{1/2}\rho' d\rho' (1/2\pi) \times {}_0\int^{2\pi} (\rho^2 - 2\rho\rho'\cos\theta' + \rho'^2)^{1/2(m-1)}\, d\theta'$$

in which n and m are nonnegative integers, P Legendre polynomial, and $0 \leq \rho \leq 1$. Differential recursion formula $1/\rho(\partial/\partial\rho + \rho\partial^2/\partial\rho^2)\, I_{n,m+2}(\rho) = (m+1)^2 I_{n,m}(\rho)$ is derived. $I_{n,m}(\rho)$ is then evaluated in two forms: first, in terms of gamma functions and hypergeometric functions, second, in terms of gamma functions and Legendre polynomials. Integral is tabulated for first few values of n and m. Albert I. Bellin, USA

2151. N. Joachim Lehmann, Contribution to the numerical solution of linear eigenvalue problems (in German, with English, French, Russian summaries), Z. angew. Math. Mech. 29, 341–356 (1949).

Author develops a method of obtaining upper and lower bounds enclosing the ith eigenvalue of the integral equation (1) $y(x) = \lambda \int K(x, s)y(s)ds$. The method consists of a modification of a maximum-minimum principle of Courant in which the $1/\lambda_i$ occur as the extrema of the integral form (2) $\int\int K(x, s)f(x)f(s)dxds$ under varying orthogonality restrictions on f, so that approximate maximization (minimization) of (2) yields lower (upper) bounds for the $1/\lambda_i$. Author's modification is based on solution of Fredholm equation $f(x) = y(x) - t_i \int K(x, s)y(s)ds$ in form $y(x) = f(x) + t_i \int R(x, s; t_i)f(s)ds$, and (2) is replaced by (3) $\int\int R(x, s; t_i)?(x)f(s)dxds$ which has the extrema $1/(\lambda_{i+j} - t_i)$ if t_i is a fixed value with $\lambda_i < t_i < \lambda_{i+1}$. Integral form (3) is transformed into a form involving kernel K only and related to an integral used by Temple. Approximate maximization (minimization) is carried out by successive approximation using method of iterated kernels. General boundary problem of Sturm-Liouville equation, the first boundary problem of $y'' + \lambda(1 + x)y = 0$ [note that y is missing in equation (60)] and integral equation $y(x) = \lambda \int |x - s| y(s)ds$ are treated as examples. Here it is often possible to start iteration with known eigenfunction of an analytically soluble, related eigenvalue problem.

Courtesy Mathematical Reviews H. O. Hartley, England

2152. Lothar Collatz, Eigenvalue problems with engineering applications (**Eigenwertaufgaben mit technischen Anwendungen**), Leipzig, Akad. Verlagsges. 1949, xvii + 466 pp.

Book is identical with author's earlier publication entitled *Eigenwertprobleme und ihre numerische Behandlung* [same source, 1945; Rev 3, **604**], except for a slight rearrangement and inclusion of an extensive new chapter called "Eigenwertaufgaben bei Matrizen." Since earlier book was rather completely reviewed only added chapter is considered here.

Chapter begins with a discussion of matrices and their latent roots, quadratic and Hermitian forms, reality of characteristic roots, together with illustrative examples chosen from mechanics. Then follows a discussion of maximal properties of characteristic numbers, transformation of principal axes, Courant's maximum-minimum principle, "Einschliessungssatz," and various numerical applications. Next comes a treatment of process of iteration, principal vectors, bounds for characteristic numbers, convergence of iteration process, and applications to systems of ordinary linear differential equations. Chapter concludes with a consideration of matrix polynomials and Cayley's equation, matrix functions and power series, approximate solutions of systems of linear equations, and special methods for setting up the characteristic equation.

Courtesy of Mathematical Reviews W. E. Milne, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 2139, 2140, 2216, 2290, 2453)

2153. O. Bottema, Some theorems from kinematics (in Dutch), Simon Stevin 27, 106–114 (1950).

By studying the motion of the plane at times $t, t + h, t + h + k$, author obtains formulas which reduce to known results in plane kinematics when h and k are both made to approach zero.

Courtesy Mathematical Reviews W. J. Nemerever, USA

2154. George Hamel, Theoretical mechanics: a systematic introduction to all branches of mechanics (in German), Berlin: Springer Verlag, 1949, xvi + 796 pp. Cloth, price 66 RM.

Textbook contains a variety of views and problems, and will be helpful to the student or engineer meeting problems not solved satisfactorily in the common texts. Although there already are available comprehensive English-language texts, Hamel gives

additional points of view on the various problems. One of main purposes of book is to present the foundations of all branches of classical mechanics in a coherent way. Reviewer's opinion is that advances can still be made by re-examining the foundations of the subject; since Hamel has devoted his life to teaching mechanics at the Imperial Institute of Technology, Berlin, and to research on the foundations of mechanics, his views are of great interest.

The text presumes the reader is well-equipped mathematically. A partial listing of comprehensive table of contents is as follows: In statics: Lagrange's principle of release, principle of virtual work. Threads, chains, wires, membranes, shells, plates; rigid, elastic, and fluid bodies. Hamilton's principle; Lagrange equations; vibrations and stability; canonical transformations. Minimum principles: least action, integral invariants, calculus of perturbations, least curvature principle of Gauss, minimum principles of theory of elasticity. Rigid body in space; Euler's equations; spinning top; problem of n-bodies. Nonholonomic systems.

O. H. Faxén, Sweden

2155. **W. Meyer zur Capellen, The path of the instantaneous pole and the Cardan position** (in German), Ingen.-Arch. 17, no. 4, 308-316 (1949).

Rauh, Marks, Bündgens and Otto [Praktische Getiebetechnik, vol. 2, VDI-Verlag (1948)] defined a Cardan position of an arbitrarily moving plane as one in which a point in moving plane traces a curve having third-order contact with an elliptic motion. Alt [Ingen.-Archiv 14, p. 319 (1944)] discussed mechanisms and their Cardan positions and obtained results different from Rauh's because he used what he considered an equivalent definition. Paper discusses these differences and shows their relation to Euler-Savary formula and inflection curve. The general four-bar linkage has no Cardan position, but there are three special four-bar linkages which do. Another analytical treatment of problem was published by Bottema [Rev 3, **1624**].

Michael Goldberg, USA

2156. **Wilhelm Blaschke, Contributions to kinematics** (in Italian), Univ. Roma, Ist. Naz. Alta Mat. Rend. Mat. e Appl. (5) 8, 268-280 (1949).

Paper reproduces two lectures concerned with a variety of problems in plane kinematics. It is first shown that Lie's contact transformation which transforms straight lines in three-dimensional space into spheres can be represented as the product of two simpler transformations, an isotropic projection and the inverse of a kinematic projection. Discussion is then turned to two-parameter families of rigid motions of one plane upon another. Such a family can be represented, in an infinity of ways, by a surface in three-dimensional space, surfaces representing same family being equivalent under the operations of a three-parameter subgroup of Lie's group of contact transformations carrying spheres into spheres. Various properties of families of motions, particularly some properties involving concepts of integral geometry, are discussed in terms of this representation. Paper closes with some brief remarks concerning relations between kinematics on a sphere and geometry of Euclidean 3-space.

Courtesy Mathematical Reviews L. A. MacColl, USA

2157. **Harold Weise, Kinematics of film transport mechanisms** (in German), Feinwerk Tech. 53: no. 4, 97-104; no. 5, 137-144 (1949).

Paper is concerned with a detailed discussion of kinematics of several types of film transport systems used in film projectors. A detailed analytical and graphical analysis of several commercial systems is developed. Curves are given of velocity of film for several types of linkage systems used in film transport mechanisms.

Louis A. Pipes, USA

2158. **C. A. Truesdell, The kinematics of vorticity,** Nav. Ord. Lab. Mem. 9591, 35 pp. (1948).

Vortex motion is presented simply as a branch of the kinematics of continuous media, without reference to any dynamical principles or to any special models. Part I is an exposition of those basic concepts and results in kinematics which are prerequisite to the analysis of vorticity. Part II introduces and interprets the vorticity field and quantities associated with it. Part III distinguishes between convection and diffusion of vorticity, discusses in detail the mechanism of convection, and closes with an exposition and critique of various proofs of Lagrange's velocity-potential theorem.

From author's summary by C. C. Lin, USA

2159. **Mario Manarini, On the stabilization of a state of equilibrium through gyrostatic actions** (in Italian), Boll. Un. Mat. Ital. 5, 56-63 (1950).

Consider a system of differential equations of the form, $\ddot{M} + G\dot{M} = \text{grad } U(M)$, where M is a vector in n-dimensional space, G is a skew symmetric matrix, U is a scalar function of M, and the dots represent differentiation with respect to t. By introducing suitable rotating coordinates, author eliminates the term in $G\dot{M}$ and thus reduces study of stability of equilibrium to Lagrange-Dirichlet theory.

D. C. Lewis, USA

2160. **Cataldo Agostinelli, On the stability of a particular motion of regular precession of an asymmetric heavy rigid body** (in Italian), Ist. Veneto Sci. Lett. Arti. 57, part II, 193-195 (1950).

Author studies characteristic exponents of a particular periodic solution of equations for motion of a heavy rigid body about a fixed point in case the center of gravity is on the axis of a circular section of ellipsoid of inertia. It turns out that the motion is generally unstable except when moments of inertia satisfy certain relations.

D. C. Lewis, USA

2161. **Bruna Fogagnolo Massaglia, On the stability of an equilibrium configuration of a system with two degrees of freedom** (in Italian), Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Nat. 83, 62-69 (1949).

Author undertakes to prove following theorem. A configuration of equilibrium of a conservative dynamical system with two degrees of freedom is unstable if the potential energy is not a minimum at configuration of equilibrium and if, of the four characteristic exponents, two are pure imaginary and two are zero. Proof involves consideration of three subcases, and treatment of one of these seems to reviewer to require further elaboration.

Courtesy Mathematical Reviews L. A. MacColl, USA

2162. **I. P. Makarov, Conditions for the approach to zero of the solutions of an inhomogeneous infinite system of differential equations** (in Russian), Doklady Akad. Nauk SSSR 68, 225-228 (1949).

System $\dot{x}_i = {}_i\Sigma_i{}^\infty\, p_{ik}(t)x_k + \varphi_i(t)$, $i = 1, 2, \ldots$, is considered where the p_{ik} satisfy the assumptions stated in a previous paper [same source, 62, 289-292 (1948); Rev 2, **158**]. The problem is to find conditions which imply that for any solution $\{x_i(t)\}$ whose initial values are sufficiently close to the origin we have $x_i(t) \to 0$ as $t \to +\infty$, $i = 1, 2, \ldots$; this question was investigated by Perron in the case of a finite system [Math. Z. 29, 129-160

(1928); 31, 159–160 (1929)]. Author proves that sufficient conditions are the existence of a λ such that

$$|\varphi_k(t)| \leqq t^{\lambda} \exp \int_{t_1}^{t} p_{kk}(t)dt$$

for $t > t_1$, and that the numbers β_{kk} of the assumptions of previous paper may be taken greater than $\lambda + 1$.

Courtesy of Mathematical Reviews J. L. Massera, Argentina

2163. Bruna Fogagnolo Massaglia, On the characteristic exponents relative to a configuration of equilibrium of a system with two degrees of freedom (in Italian), Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Nat. 83, 70–81 (1949).

A configuration of equilibrium of a dynamical system with two degrees of freedom is stable if four characteristic exponents have negative real parts. There is a set of known conditions, due to Hurwitz, which is necessary and sufficient for the characteristic exponents to have this property. Author states that Hurwitz conditions generally lead to rather complicated calculations, and seeks a simpler set of conditions which shall be at least sufficient for the configuration of equilibrium to be stable. He finds three such sets of conditions, one of which is already known. The sets of conditions are of same general character as Hurwitz conditions, but they lend themselves more readily to physical application and interpretation.

Courtesy Mathematical Reviews L. A. MacColl, USA

2164. Henri Pailloux, On certain nonholonomic systems (in French), C. R. Acad. Sci. Paris 230, 1501–1504 (1950).

Systems referred to are those whose constraint equations are nonlinear in velocities and have the property of being "perfect," i.e., the forces of constraint do no work in any virtual displacement compatible with the restraints. In such cases, equations of motion are much simpler than in the general case. Linear case is excluded, apparently because a linear perfect system is essentially holonomic. D. C. Lewis, USA

2165. Jakob Tjakko Groenman, Investigations on the three-bar curve by means of circular coordinates (in Dutch, with English summary), Thesis Technische Hogeschool te Delft, 104 pp. (1950).

Monograph on the three-bar curve deals particularly with the properties which have been found since 1920 by Hippisley, Bennett, Morley, Weiss, Alt, Mayer, and others. Author has chosen analytical treatment developed by Haarbleicher, who uses isotropic lines through origin as axes of coordinate system. Much attention is given to theorems on foci and nodes and to the fact that the curve is invariant for certain isogonal transformations. Special systems of three-bar curves are considered, such as system of curves with common foci and nodes. Finally, theory is applied to some cases in which the foci and nodes have a particular position such as curve with three cusps and that in which foci and nodes coincide. In this chapter author gets some new results.

Courtesy Mathematical Reviews O. Bottema, Netherlands

2166. R. Grammel, On the computation of the Poinsot motion (in German), Ingen.-Arch. 18, 53–59 (1950).

Differential equations for motion of a rigid body about a fixed point under influence of no external forces can be integrated by quadratures and also admit the elegant geometric interpretation of Poinsot. Author modifies the known solution in terms of elliptic functions to obtain approximate formulas more suitable for computational purposes. This is done by discarding all but first terms in the Fourier series of certain theta functions.

D. C. Lewis, USA

2167. Carlo Tolotti, On the impulsion motion of an holonomic system subjected to several simultaneous one-sided constraints (in Italian), Ann. Mat. Pura Appl. 29, 251–257 (1949).

In elementary mechanics, motions involving two colliding bodies are studied with the help of the Newtonian hypothesis to the effect that ratio of relative velocity just after collision to relative velocity just before collision is a constant (coefficient of restitution). Author generalizes resulting equations to apply to general holonomic dynamical systems subjected to several simultaneous one-sided constraints. Proof of the final equations is based on Newtonian hypothesis as well as on assumptions about types of admissible one-sided constraints. D. C. Lewis, USA

2168. Cataldo Agostinelli, On motion about a fixed point of a heavy rigid body whose center of gravity lies on axis of one of the circular sections of the ellipsoid of inertia (in Italian), Ann. Mat. Pura Appl. 30, 211–224 (1949).

Author proves existence of certain motions, depending on five constants of integration, which are expressed parametrically by means of certain power series. Recursion formulas for coefficients of these series are explicitly given. When moments of inertia satisfy a certain relationship, these series reduce to polynomials, at least for certain values of the integration constants.

D. C. Lewis, USA

2169. Giorgio Sestini, Motions of rigid body with one point sliding on a smooth line (in Italian), Atti. Sem. Mat. Fis. Univ. Modena 3, 18–28 (1949).

Suppose the motion of a rigid body about a fixed point P under action of given forces is known. Author proposes to determine the lines along which it is possible for P to move without friction and without disturbing the known rotatory motion of body about P. D. C. Lewis, USA

2170. Tristano Manacorda, On the motion of an asymmetric rigid body about a fixed point (in Italian), Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. 6, no. 6, 711–714 (1949).

Conditions are obtained in order that a rigid body, with center of gravity G and with fixed point O, should move in such a way that the component of moment of momentum in direction OG should vanish when moment of applied forces also has a vanishing component in direction OG. D. C. Lewis, USA

2171. L. Castoldi, Lagrange equations for dynamic systems in servomotorial constraints, Nuovo Cim. 6, 180–186 (1949).

Purpose of paper is to define a "servomotorial" type of constraint in contrast to usual "kinematic" type. General distinction is not clear, but author's discussion suggests following interesting reflections.

A fixed constraint is usually assumed to be maintained by a linkage or its equivalent. Same constraint, however, can be maintained by an ideal servomechanism (in which hunting is completely suppressed); motion will then be different in features that are not controlled by constraint. Author cites as an example the spherical pendulum, whose length l might be caused to vary as a function $f(\Theta)$, Θ denoting deflection from the vertical, either by means of a linkage or by a servomotor, which might raise or lower supporting cord in proper manner through a fixed eye.

With type of constraint so chosen that constraint forces contribute only to generalized force for one coordinate, these forces can be ignored in writing down Lagrange's equations for other coordinates. Thus, if l, Θ, φ are coordinates for spherical pendulum, in writing the equations for Θ and φ, constraint forces may be ignored if the constraint is by servo in the manner described (a servomotorial constraint), but not if constraint is

kinematic as by usual linkage or smooth, fixed surface on which the bob may slide. E. H. Kennard, USA

2172. Iv. **Tzenoff, Some new forms of the general differential equations of motion for material systems** (in French), C. R. Acad. Bulgare Sci. Math. Nat. 2, no. 1, 13–16 (1949).

The reader may readily verify the indentity,

$$-\frac{\partial T}{\partial x_i} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) - \frac{\partial \dot{T}}{\partial \dot{x}_i}$$

where T is a function of $x_1, \ldots, x_n, \dot{x}_1, \ldots, \dot{x}_n$ and

$$T = \frac{dT}{dt} = \sum\nolimits_{i=1}^{n}\left(\frac{\partial T}{\partial \dot{x}_i}\ddot{x}_i + \frac{\partial T}{\partial x_i}\dot{x}_i\right).$$

Hence Lagrange's equations may be written in the rather unfamiliar form

$$2\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) - \frac{\partial \dot{T}}{\partial \dot{x}_i} = Q_i$$

This and similar identities are considered by author, leading to novel forms for equations of motion of both holonomic systems.

D. C. Lewis, USA

2173. **C. D. Strang and C. R. Lewis, On the magnitude of the mechanical component of solid friction,** J. appl. Phys. 20, 1164–1167 (Dec. 1949).

Article reports results of an experimental study of dry, kinetic friction which was designed to determine per cent of coefficient due to work of lifting irregularities of one surface over those of the other, i.e., coulomb friction. It was found that if instrumentation was capable of resolving normal movements of surfaces, coulomb friction contributes only a small per cent of the total. However, it was stated that the possible existence of motions which could not be measured with experimental ensemble used prevented results from being fully conclusive.

R. G. Wilson, USA

2174. **Cataldo Agostinelli, Canonic equations of motion of an anolonomic system as a system associated with a certain Pfaffian** (in Italian), Boll. Un. Mat. Ital. 4, 345–348 (1949).

It is shown that differential equations of motion of a nonholonomic system, reduced to canonical form, coincide with system of equations associated with a certain Pfaffian.

E. Leimanis, Canada

2175. **Luigi Castoldi, Lagrangian equations for systems with anolonomic bonds with nonlinear velocity** (in Italian), Ist. Lombardo Sci. Lett Rend. Cl. Sci. Mat. Nat. 11, 189–200 (1949).

Equations described in title are of following form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i + \sum\nolimits_{j=1}^{m}\lambda_j\frac{\partial \varphi_j}{\partial \dot{q}_i},\ i = 1, \ldots, n$$

where the m constraint equations are $_j(\dot{q}, q, t) = 0$. Author also transforms the equations by introducing so-called kinetic characteristic parameters. D. C. Lewis, USA

Gyroscopics, Governors, Servos

(See also Rev. 2463)

2176. **N. N. Bautin, Theory of an escapement regulator with spring blade** (in Russian), Doklady Akad. Nauk SSSR 72, 19–22 (1950).

In various mechanisms, among them those used to measure very short time intervals, there occur systems such as considered here. The basic differential equations between consecutive impulses are $M\ddot{y} = P$, $m\ddot{x} = -k_0x - b\dot{x}$ where the coefficients are all constant. Each impulse yields a linear transformation Ω on the values of $x, \dot{x}, y$, "before" and "after," where Ω depends on various parameters of system. Fixed points of Ω determine the periodic regimes. Their determination is reduced to solution of a certain transcendental equation. In absence of friction, equation becomes a quadratic whose unique positive root determines the periodic regime, and it may be stable or unstable. When there is friction there may be present many periodic regimes, some stable, some unstable. As many as four stable periodic regimes have actually been verified experimentally with simple models.

S. Lefschetz, USA

2177. **J. Daniel, Governor speed-droop and its elimination** (in French), Houille blanche 5, 15–31 (Jan-Feb. 1950).

In first part author shows that a static governing can be carried out by introducing the time integral or derivative of controlled value; in this case stability is reached by adjusting "slowness of governing." Another possible method is static governing around average value adjusted by means of another apparatus (wattmeter); in this way, a "virtual speed-droop" is introduced. In second part author shows that use of mentioned integrator in governing of hydraulic turbines improves stability. He compares this method with another one aimed at the same objective.

Miroslav Nechleba, Czechoslovakia

✎2178. **Winfried Oppelt, Continuous regulation processes (Stetige Regelvorgänge)**, Hannover, Wissenschaftliche Verlagsansalt K. G., 1949, 144 pp., 42 figs. Paper, 6 × 8 in., price unstated.

Book follows another, entitled *Regulation Fundamentals*, which reviewer has not seen. Both are concerned with regulators and control systems, and make only passing mention of servomechanisms. However, they analyze closed-loop regulation by means of the same techniques which are presented in standard American books on servomechanisms. First book is concerned with theory, the present one with applications.

Book should interest all regulation engineers, not that it contains any point of view or information unknown on this side, but because of enormous number of different systems which it manages to analyze in a very small compass. For instance (and this is only part of contents), author presents no less than 41 pages of diagrams, each page concerned with analysis of a system in which machine and regulator are characterized by their respective differential equations (with constant coefficients). Each page contains eight diagrams, four of them giving several loci each in (Nyquist) complex-frequency plane, corresponding to different assumptions, and the four diagrams opposite giving corresponding time variations (transients) of the control variable or of output variable. There are tables of examples of each type of system, taken from all types of fields, thermic, hydraulic, pneumatic, mechanical, and electrical. This should be of considerable value to student, instructor, and designer of regulation systems.

P. Le Corbeiller, USA

2179. **Aaron S. Bosenbom and Richard Hood, General algebraic method applied to control analysis of complex engine types,** Nat. adv. Comm. Aero. tech. Note 1908, 42 pp. (July 1949).

Paper presents a general algebraic method employing operational functions to describe engine and control characteristics that can be utilized in solution of control problems of complex gas-turbine engine types. Use of method permits definition of

control characteristics required to produce noninteraction among controlled variables, and to give any desired response action.

Algebraic method is applicable to control of any process or device which has more than one input and more than one output, outputs each being functions of several inputs. Process is controlled by automatic manipulation of inputs in accordance with deviations of outputs from set values. Paper shows how to combine deviations in such a way that if any one output has a deviation, all inputs are controlled in proper proportion so that no other outputs are disturbed; this is defined as noninteraction.

Paper represents first step in a very important field of automatic control. Herbert Harris, Jr., USA

2180. **Melvin E. LaVerne and Aaron S. Bosenbom, Methods determining frequency response of engines and control systems from transient data,** Nat. adv. Comm. Aero. tech. Note 1395, 29 pp. (Aug. 1949).

Paper derives and presents standard equations of Laplace transformation used for obtaining frequency response of a linear system from general transient data. Some useful variations of equations are derived to permit transients to be constant or sinusoidally oscillating in steady state. Also presented are equations which allow use of straight line approximations of time function for simplification of calculations.

Three examples of systems are given in which frequency response is obtained using Henrici rolling sphere harmonic analyzer. First two are elementary to show correlation of calculated frequency response with that obtained using analyzer. Third example, using experimental transient data from a turbine propeller engine, empirically shows transfer function from blade angle to engine speed to be a first-order (single time constant) system.

Report is clearly written, but more experimental data from actual and varied systems, with indication of how described analysis techniques are applied, would be more valuable than summary of equations presented. W. R. Ahrendt, USA

2181. **N. A. Železcov, Vyšnegradskiĭ diagrams for an isodromic regulator with nonlinear action** (in Russian), Avtomatika i Telemehanika 10, 424–436 (July 1949).

A Vyšnegradskiĭ diagram for a mechanism is a chart showing for what values of the parameters of the mechanism the linearized equations governing the device have only stable solutions. Paper is a detailed discussion of the stability of a particular regulating system.

Courtesy Mathematical Reviews E. N. Gilbert, USA

2182. **H. Wormsbecher, Precession phenomena with spinning pot motors** (in Dutch, with English summaries), Ingenieur 's-Gravenhage 61, E 51–56 (July 1949).

Self-sustained precession motion of a disk or pot on rapidly rotating elastic spindle is explained through mechanical hysteresis of spindle material. Unbalance gives spindle a stationary deflection, hysteresis is eliminated, and precession prevented. Calculated values for frequency of precession are confirmed by experiment for a disk. R. Vermeulen, Holland

2183. **Rene Feiss, Modern control dynamics and stability criteria as applied to gyroscopic speed detectors with hydraulic drive,** Trans. Amer. Soc. mech. Engrs. 71, 979–988 (Nov. 1949).

A speed detector, using precession of a gyroscope, is considered as a regulator for a turbine. A model is described and results of tests are cited. A frequency response method of determining stability of an automatic regulating system using gyroscopic rate detector as sensing element is outlined and results shown in form of curves. Unfortunately, modern methods were not applied so that results are somewhat difficult to interpret.

R. G. Wilson, USA

2184. **Konrad Beyerle, Gyroscopic equipment** (in German), Naturf. Mediz in Deutschl. 1939–1946, 7, 211–233 (1948).

A discussion of different kinds of gyroscopic equipment. Treatment is largely of an engineering rather than of a mathematical nature. D. C. Lewis, USA

Vibrations, Balancing

(See also Revs. 2144, 2150, 2178, 2182, 2228, 2453)

2185. **J. J. Stoker, Nonlinear vibrations in mechanical and electrical systems,** New York: Interscience Publishers, Inc.; London: Interscience Publishers Ltd., 1950, xx + 273 pp. Cloth, 5.3 × 7.7 in., $5.

Prof. Stoker's book is recommended to all who are interested in a systematic study of this subject. His presentation renders it particularly attractive to physicists and engineers owing to large number of examples illustrating various mathematical concepts such as singularities, limit cycles, etc., which thus acquire an almost intuitive aspect. On the other hand, once this intuitive approach has been sufficiently prepared, author proceeds with necessary rigor to prove existence theorems for these concepts introduced more or less on physical grounds.

Chapter I is a short summary of theory of linear vibrations for a system with one degree of freedom; it serves both as a reference and contrast for the nonlinear theory. Chapter II introduces the reader to simplest case of nonlinear problems associated with behavior of nonlinear conservative systems. Phase-plane representation of integral curves is introduced in this chapter and is illustrated by many examples. Chapter III is also of a rather elementary character, and deals with differential equations of a more general type, namely, those which, in addition to nonlinear restoring force, possess also a nonlinear damping. The end of chapter contains a number of physical examples illustrating the concept of singular points as various types of equilibria of dynamical systems. Chapter IV is of a less elementary character, and deals with the so-called Duffing equation with forcing term. This subject is of interest mostly in mechanical problems, and is treated by two different methods (iteration and perturbation). Chapter V is perhaps the most important of the whole book, and is split into parts A and B. Part A deals with self-excited systems. In this chapter author introduces fundamental concept of limit cycles on physical grounds, but a fuller mathematical treatment is given in appendices III and VI. Second half of part A is devoted to so-called relaxation oscillations, and contains a considerable amount of original work done by author, who contributed much to recent studies of this subject. Part B deals with forced oscillations in self-sustained systems. After a brief review of earlier work of van der Pol, author gives an interesting and detailed presentation of Andronow-Witt method. This is perhaps the most interesting part of the book. Chapter VI is devoted to questions of stability. Author reduces this problem to analysis of local behavior of variational equations (of Hill type) in various parts of response curves, and applies this procedure to Duffing equation. N. Minorsky, France

2186. **A. M. Schulte, A slight improvement of Southwell's method for the approximative computation of the lowest frequency of a homogeneous membrane,** Appl. sci. Research A. 2, 93–96 (1950).

Southwell uses a numerical formula analogous to Rayleigh's quotient for the approximate computation of the lowest fre-

quency. Schulte proposes a slight modification in process and states that new method reduces computational work to about 80% of work needed formerly.
Courtesy Mathematical Reviews W. E. Milne, USA

2187. **W. F. King, A non-seismic torsiograph,** Soc. exp. Stress Anal. 7, 41–44 (1950).

2188. **A. G. Pugsley, On the natural frequencies of suspension chains,** Quart. J. Mech. appl. Math. II, 412–418 (Dec. 1949).

Author derives a simple expression for natural frequency of a suspension chain. He shows that natural frequencies are independent of mass per unit length of chain, and practically independent of span. They vary approximately as square root of span, and first three are roughly in ratio of 1.0, 1.5, and 2.0. Results are arrived at from a combination of experimental work, dimensional analysis, and elementary theory. Paper confirms work of Hans Reissner [J. appl. Mech. 10, 23–32, (1943)].
R. M. Wingren, USA

2189. **Edwin T. Kruszewski, Effect of transverse shear and rotary inertia on the natural frequency of a uniform beam,** Nat. adv. Comm. Aero. tech. Note 1909, 15 pp. (July 1949).

Author gives boundary conditions and finds frequency equations for cases of cantilever and free-free beams, using differential equation given by Timoshenko. Numerical calculations performed show that a large reduction in the natural frequencies is possible, particularly for higher modes, when rotatory inertia and shear effects are considered. N. O. Myklestad, USA

2190. **W. Wuest, Bending-vibrations of curved rods and pipes clamped on one end** (in German), Ingen.-Arch. XVII, no. 4, 265–272 (1949).

Frequency equation for vibrations of curved rods, originally due to R. Hoppe [J. Math. (Crelle), 73 (1871)] is used for a circular rod with one end fixed. Rod is assumed to undergo an axial extension. Particular case of a light rod with a point load is treated in detail and numerical values are obtained for the frequencies and angular displacement. General solution is discussed with help of a quantity k, a constant multiplier of frequency, whose limiting values are found to be 0, 0.11340 and 17.6366. A number of numerical results are given showing how they may be used in practice. B. R. Seth, USA

2191. **Walter Mudrak, Determination of natural frequencies of continuous beams and frames** (in German), Z. angew. Math. Mech. 28, 258–263 (Sept. 1948).

A practical method for computation of four lowest natural frequencies of continuous beams and uniplanar frames is developed and illustrated by numerical examples. Basis of method is a nomogram giving natural frequencies of a uniformly loaded beam of constant cross section, simply supported at both ends except for elastic restraints against rotation. A method of successive approximations extends usefulness of diagram to cover subject problem. Only slide-rule computations are required. In case of other than very simple frames, no indication is given that method always converges, or that it will converge to a particular eigenvalue. Simplicity of procedure makes it of interest to engineers who encounter determination of natural frequencies of frames as a design problem. Lawrence E. Goodman, USA

2192. **H. A. Kramers, Vibrations of a gas column,** Physica 15, 971–984 (Dec. 1949).

Author starts with hydrodynamical equations in differential form for gas flow in a cylindrical tube and reviews work of Kirchhoff and Taconis [same source, p. 733 (1949)] for vibrations in gas column. He then obtains a solution which takes into account existence of a temperature gradient along tube, influence of heat conductivity, and effects of internal friction of gas, whereas this last was neglected by Taconis. Author assumes small disturbances from a steady condition, and this assumption of small vibration amplitudes leads to a solution which does not give spontaneous vibrations so often observed. Damping of oscillations *is* lowered, but full explanation of observed instability seems to lie in nonlinear effects not included by author.
W. C. Johnson, USA

2193. **B. R. Dudley and H. W. Swift, Frictional relaxation oscillations,** Phil. Mag. 40, 849–861 (Aug. 1949).

Authors consider a graphical method for solving the nonlinear equation $m\ddot{x} + c\dot{x} + kx = F(\dot{x} - u)$ where m, c, k and u are constants and F is an empirical function of $\dot{x} - u$ (notation is reviewer's). Equation represents motion of a mass, spring, dashpot system with friction between mass and a surface moving with velocity u. Motion is investigated for several typical forms of function F as found by experiment and solution is presented in the $(\dot{x}, x)$ plane. Under certain conditions, oscillations are periodic with part of motion "stick-slip," i.e., mass adheres to moving surface for part of cycle due to static friction F (0). Under other conditions, motion either decays or grows to asymptotic values. Analysis is an extension of work of Thomas (1930) and Blok (1940) and has been stimulated by recent experiments to measure coefficient of dynamic friction. Albert I. Bellin, USA

2194. **J. C. Georgian, Torsional viscous-friction dampers,** Trans. Amer. Soc. mech. Engrs. 71, 389–399 (1949).

Author discusses properties of new silicon oils which can be used to advantage in Lanchester dampers. Further, author gives design calculation details, a numerical example, and manufacturing hints which make paper especially useful for designers. Appendix gives the theory of the damper as applied to a multimass system. A discussion by G. J. Dashefsky, B. E. O'Connor and author focuses attention on history of theory and construction, and deals with some details of computation.
W. L. Esmeijer, Holland

2195. **Walter W. Soroka, Note on the relations between viscous and structural damping coefficients,** J. aero. Sci. 16, 409–410 and 448 (July 1949).

Structural damping is defined as opposite in phase to velocity, but proportional in magnitude to amplitude and not to velocity. Paper shows that natural frequency is *raised* by structural damping in about the same degree as it is lowered by viscous damping. J. P. Den Hartog, USA

2196. **J. D. Eshelby, Dislocations as a cause of mechanical damping in metals,** Proc. roy. Soc. Lond. Ser. A, p. 197 (June 1949).

Author extends theory of C. Zener concerning dissipation of energy in vibrating bodies through thermal currents caused by time-varying stress gradients. He assumes stress gradients considered by Zener to be caused only by time-varying surface tractions. Author suggests that dislocations within vibrating bodies may also cause stress gradients which lead to energy dissipation through thermal currents. In the analysis all dislocations are assumed to be parallel and of infinite length. Mathematical details are given in form of six appendixes. There is discrepancy between theory and what sparse experimental data are available concerning dependence of damping on frequency. Regardless of this, reviewer believes that suggestive ideas presented in this

complicated but technically important subject are worthy of further analysis. W. H. Hoppmann, II, USA

2197. F. K. Rubbert, On vibrations with combined damping (in German), Ingen.-Arch. 17, no. 3, 165–166 (1949).

Formulas are derived for viscous and Coulomb damping constants in terms of initial, n^{th}, and $2n^{th}$ amplitudes of free vibration. Reviewer agrees with author's statement that his formulas are more advantageous for determining damping constants from a vibration record than those given by Bogel [same source, 12, p. 247 (1941)]. R. P. Felgar, Jr., USA

2198. Jenö Egerváry, Application of Rayleigh's method to the determination of the critical speed of a rotating system (in Hungarian, with Russian and English summaries), Mat. Lapok 1, 16–26 (1949).

Author explains Rayleigh's method by applying it to longitudinal vibrations of a system consisting of a beam clamped at one end and carrying a concentrated mass at other end. At first, mass of beam is neglected in comparison with concentrated mass, and a correction of this first approximation for lowest characteristic frequency of system is obtained from a consideration of kinetic and potential energies. Principal problem discussed concerns a system of importance in turbine manufacture: a rotating shaft carrying a heavy cylinder (rotor). Diameter of the shaft may be staggered, but the whole system is assumed to be symmetric with respect to a plane perpendicular to shaft. Author writes down partial differential equations governing oscillations of this system, together with transition and boundary conditions; and after neglecting mass of shaft, he solves (exactly) this boundary-value problem. The transcendental equation obtained for lowest characteristic frequency has only two essential parameters in it and hence a nomogram could be (but is not in paper) constructed for its solution.

Courtesy Mathematical Reviews A. Erdélyi, USA

2199. B. A. Ryabov, Determination of the parameters of the régime of established auto-oscillation of certain systems (in Russian), Doklady Akad. Nauk SSSR 71, 663–666 (1950).

In connection with the oscillation of a certain system, the following system of equations is encountered:

$$\sum_{k=1}^{n} P_{kj}(p)x_k = \delta A_j\sigma_0,$$

where $\delta = 1$ for $x_\sigma \geqq x_{\sigma 0}$, $\dot{x}_\sigma \geqq 0$, $\delta = -1$ for $x_\sigma \leqq -x_{\sigma 0}$, $\dot{x}_\sigma \leqq 0$. Here $p = d/dt$, $P_{kj} = a_{kj}p^2 + b_{kj}p + c_{kj}$; a_{kj}, b_{kj}, c_{kj}, A_j and σ_0 are constants. Author shows how to solve this system in terms of solution of the linear homogeneous equation.

R. Bellman, USA

2200. N. Minorsky, Self excited oscillations in systems possessing retarded actions, Proc. seventh int. Congr. appl. Mech. 4, 43–51 (1948).

In first part of paper author reminds his readers that *linear* differential equations with terms representing retarded action, e.g.,

$$\ddot{x}(t) + p\dot{x}(t) + \rho x(t + h) = 0 \qquad [1]$$

have particular integrals $x = e^{zt}$, where z is determined by the transcendental equation

$$z^2 + pz + \rho e^{-hz} = 0 \qquad [1a]$$

and he recalls how these equations may be treated. Then he proceeds to equations where a (negative) cubic term is added, e.g.,

$$\ddot{x}(t) + p\dot{x}(t) + \rho x(t + h) - \rho_3 x^3(t + h) = 0 \qquad [2]$$

This equation is treated by a procedure which is standard (Kryloff-Bogolyuboff) for nonlinear differential equations of ordinary type. It is assumed that equation [2] can be solved by an expression $x = a \sin(\omega t + \beta)$, where a and ω are slowly varying functions of time, and furthermore that

$$x(t + h) = a \sin(\omega t + \beta - \phi) \qquad [3]$$

which means that the retardation only affects phase but not amplitude a.

Analysis leads to a satisfactory value for the amplitude

$$a_{st} = (4\sigma/3\rho_3 \sin\phi)^{1/2}$$

where $\sigma = \rho \sin\phi - p\omega$. K. Klotter, USA

2201. D. Williams, Displacements of a linear elastic system under a given transient load, Aero. Quart. 1, 123–136 (Aug. 1949).

To find displacements and stresses by standard procedure one has to expand the given transient load in a series of normal modes. Many terms in this series have to be considered if a close enough approximation is to be obtained. If there is a concentrated load, number of terms necessary is likely to be quite large.

Author's method consists in dividing the forces acting on body into two groups: given loads and inertia forces. Total displacements and stresses at every instant are then identical with *static* displacements and stresses under those two groups of forces. First is already known. Second group (inertia forces) may readily be calculated with aid of natural frequencies and normal modes of system. Advantage of procedure lies in fact that in general only inertia forces belonging to lowest natural frequencies are of importance and have to be taken into consideration. Method is also applicable to complicated systems where natural frequencies and normal modes may be determined experimentally.

Heinz Parkus, Austria

Wave Motion, Impact

(See also Revs. 2222, 2229, 2350, 2366, 2391, 2452, 2517, 2518)

2202. H. A. Rahmatulin, Investigation of the laws of propagation of plane elastic-plastic waves in a medium with a variable elastic limit (in Russian), Prikl. Mat. Mekh. 14, 65–74 (1950).

Plastic deformation theory is used to discuss propagation of plane longitudinal waves in a medium in which the elastic limit $\sigma(x)$ varies as a function of the coordinate. It is shown that the law of propagation of a plastic wave depends only on $\sigma(x)$ and the stress-strain curve and is independent of the manner of loading. In general, three stressed regions will exist in such medium: an elastic region, a region of plastic loading, and a region of unloading. For the case of linear hardening, general solutions of the wave equation are given for each region in media with monotone varying elastic limit. Composite solutions are discussed for nonmonotone cases. As an example, the elastic limit is determined as a function of length in an originally homogeneous isotropic elastic rod which is dropped on a rigid plate.

Courtesy Mathematical Reviews H. I. Ansoff, USA

2203. P. E. Duwez, D. S. Clark, and H. F. Bohnenblust, The behavior of long beams under impact loading, J. appl. Mech. 17, 27–34 (1950).

Plane cross sections are assumed to remain plane, rotary inertia and shear deformation are neglected and the impact is supposed to impart a constant velocity to a single cross section of an infinite beam. Plastic action predicted by ordinary beam theory is taken into account for two cases: ideal plasticity, and linear strain

hardening. However, local plastic deformation at the impact surface is neglected. It is found that in the plastic as in the elastic case, y/t is a function of x^2/t where y is the beam deflection at a distance x from the point of impact at time t. Comparison is made between experiments on steel and copper beams and deflection predicted by theory. Agreement is good but the critical check of strain was not made. [See Dohrenwend, Drucker, and Moore, Proc. Soc. exper. Stress Anal. 1, no. 2, 1–10 (1944)].

D. C. Drucker, USA

2204. **Paul Bergeron, The complexity of water-hammer phenomena in pump installations and an endeavour to classify the general remedial solutions** (in French), Houille blanche, no. spec. B, 724–732 (1949).

Using graphical methods, author analyzes difficulties arising in discharge line of such systems on stopping of pump. These difficulties are caused by initial low-pressure wave that produces cavitation and by its reflection in form of a high-pressure impact wave. Solution of difficulties by using devices such as air reservoirs, water columns, water reservoirs, one-way valves, pump bypass lines with check valves, flywheel on pump, and others are briefly discussed. Each device has its useful range and its limitations.

Alexander Yorgiadis, USA

2205. **J. T. Muller, Measuring shock with high speed motion pictures,** Nav. Res. Lab. Rep. 5-2376, 15–22 (1948).

Paper briefly describes and illustrates operation of high-speed motion picture camera, preparation of test object, and precautions to be considered when films are taken. Difficulties encountered in obtaining numerical data as well as advantages and disadvantages of high-speed motion picture film for shock work are enumerated.

From author's summary

2206. **J. L. Jones, Piezoelectric accelerometers,** Nav. Res. Lab. Rep. 5-3276, 54–60 (1948).

Basic principles of the mass-spring accelerometer system are discussed in relation to phenomenon of piezoelectricity which is used to convert acceleration effects to electrical transients. Advantages and limitations of crystal pickups are discussed, and influence of these characteristics on the instrumentation techniques employed is shown. A few applications of corrected and uncorrected crystal accelerometer circuits at NOL are discussed in relation to type of acceleration pattern studied.

From author's summary

Elasticity Theory

(See also Revs. 2152, 2202, 2269, 2299, 2302, 2315)

2207. **On the application of oblique coordinates to problems of plane elasticity and swept-back wing structures,** Coll. Aeron. Cranfield. Rep. 31, 37 pp. (Jan. 1950).

Object of report is twofold. On the mathematical side, it seeks to illustrate use of oblique coordinates in applications to elasticity and structure theory. On the practical side, it seeks to provide methods by which designers can solve problems of stress distribution and deflection for the case of sweptback wing structures, whose ribs lie parallel to the direction of flight.

Report is divided into three parts. In part I, the mathematical basis is developed. Formulas are derived which express fundamental concepts and relations of geometry, kinetics, statics and plane elasticity in terms of vector components in oblique coordinates. In part II, results obtained in part I are applied to a uniform, symmetrical rectangular section, sweptback box. In part III, main results of part II are generalized to cover case of a more representative wing structure. This represents an extension of usual engineer's theory of bending and torsion to cover case of sweptback wings with ribs parallel to flight direction.

E. Reissner, USA

2208. **A. Wiszniewski, Study of elastic deformations on free surface of a structural member by measuring linear dilation of the sides of an equilateral triangular element** (in French), L'Ossature métallique 14, 362–369 (July-Aug. 1949).

Paper gives an analytical and graphical analysis of "delta" strain rosette problem. Analytical analysis is conventional. Graphical construction appears unnecessarily complicated to reviewer (perhaps because of unfamiliarity).

Horace J. Grover, USA

2209. **W. Olszak, Plane strain and plane stress systems,** Arch. Mech. Stos. Danzig 1, 255–263 (1949).

It is known that the solution of generalized plane stress problems can be deduced from the corresponding solution for plane strain by changing the Poisson ratio σ into the plane stress ratio σ_1 such that $(1 + \sigma)(1 - \sigma_1) = 1$. By making a function of z the axial coordinate, stress-strain relations are generalized and plane strain and generalized plane stress problems become limiting cases. The corresponding components of displacements are obtained and the case of a single force acting within an infinite plate is discussed in detail.

B. R. Seth, USA

2210. **P. Lardy, Theory of elasticity for the parallelogram-shaped plate** (in German), Schweiz. Bauztg. 67, 419–422 (July 1949).

Author deduces fundamental equations of elasticity governing generalized plane stress, when coordinates are oblique cartesian coordinates. A number of problems are solved by use of these equations. One problem deals with a plate whose edges form a parallelogram, under a load distributed uniformly along two opposite edges. Another problem deals with a half plane under action of a periodic load along edge, load making a constant angle with edge.

G. E. Hay, USA

2211. **Masakazu Higuchi. On the two-dimensional stress in aeolotropic bodies,** Kyushu Univ. Res. Inst. elast. Engng. Rep. no. 4, 4, 47–71 (1947).

Paper examines stress distributions in an infinite aeolotropic plate containing an elliptical hole or rigid plug. In special case of a circular hole in a plate under tension, hoop stress is evaluated numerically for wood permeated with phenol resin, elastic constants for such wood being found by author's experiments.

Problem of a semi-infinite aeolotropic plate under unit normal tensile force on part of straight edge is also discussed.

Most problems considered by author have also been solved by other writers. See: S. Lekhnitskiĭ: Prikl. Mat. Mekh. 3, 56–69 (1936); C. R. Acad. Sci. USSR 4, p. 3 (1936); N.S.1, 77–90 (1937); 2, 181–210 (1938); 5, 71–91 (1941). D. Sherman, ibid. 9, 347–352 (1945). C. B. Smith, U. S. Dept. Agric. For. Prod. Lab. 1510 (1944). A. E. Green and G. I. Taylor, Proc. roy. Soc. 173, 162–173 (1939). A. E. Green, ibid. 173, 173–192 (1939); 180, 173–208 (1942); 184, 231–252 (1945); 184, 301–345 (1945). G. H. Livens and R. M. Morris, Phil. Mag. 38, 153–179 (1947).

A. E. Green, England

2212. **R. S. Rivlin, Large elastic deformations of isotropic materials VI. Further results in the theory of torsion, shear and flexure,** Phil. Trans. roy. Soc. Lond. Ser. A 242, no. 845, 173–195 (Dec. 1949).

To impressive list of exact solutions of equations for finite strain of an incompressible isotropically elastic material already obtained by author [same source: 241, 379–397 (1948), see REV 2, **1491;** 195, 463–473 (1949), see REV 2, **1492;** Proc. Camb. phil. Soc. 45, 485–487 (1949), see REV 3, **652**] are added three more. As in previous solutions, results are valid for *any* form of strain energy, and, thus, are definitive contributions to theory of incompressible elastic materials. Solution 1: A circular cylindrical tube is uniformly extended, then uniformly inflated at constant length, then uniformly twisted with length and diameters held constant. Special cases: cylindrical rod, circular cylindrical tube turned inside out, torsion and extension without inflation. Solution 2: Deformation begins as for solution 1, but instead of torsion there is now first a simple (circular) shear in plane of cross section, second a simple (cylindrical) shear along axis. Special case: The two shearings without extension or inflation. Solution 3: A rectangular parallelopiped is flexed into a sector of a right circular cylindrical tube bounded by two axial planes and by two planes normal to axis. (This solution is a generalization of that given in Proc. roy. Soc. quoted above.) Solutions present the resulting displacement distributions and surface loadings required to produce them. They are specialized to the case of a Mooney material, whose strain energy has a particularly simple form containing only two elastic moduli. C. A. Truesdell, USA

2213. W. Zerna, The membrane theory of shells of revolution, (in German), Ingen.-Arch. 17, no. 3, 223–232 (1949).

Analysis is given of behavior of a thin elastic shell under applied load. Shell is treated as a membrane, i.e., state of stress is regarded as independent of displacements, and attention is thus given only to the direct and shear stress resultants in the middle surface, which is taken to be an arbitrary surface of revolution. Position of a point in the middle surface is specified in terms of a parametric coordinate ϑ and an angular coordinate φ ($0 \leq \varphi \leq 2\pi$). Equations of equilibrium are presented in a simplified form through introduction of suitably modified stress resultants, and it is then proved that, if the direct stress resultant in a longitudinal direction N_ϑ is known, then other two stress resultants are found simply. The determination of N_ϑ depends upon solution of a linear, second-order, nonhomogeneous, partial differential equation in ϑ and φ whose type varies with sign of the Gauss curvature of middle surface. Components of applied load and also stress resultants are next expressed by Fourier series in φ with coefficients dependent upon ϑ. Solution of problem is then proved to be dependent upon the particular integrals of a set of linear, second-order, nonhomogeneous, ordinary differential equations in ϑ. These equations have variable coefficients, and do not appear to admit, in general, of simple solutions; attention is therefore directed toward an approximate method of integration of the equations, and this involves consideration of the corresponding integral and variational equations. The approximate method of integration is illustrated for a particular shape of shell and a particular type of applied load, and some discussion of convergence is given.

P. Neményi and C. Truesdell have given [Proc. nat. Acad. Sci. Wash. 29, p. 159 (1943)] a method of exact solution of membrane equations for any surface of revolution and for any applied load representable by a Fourier polynomial; extensions of that work are also referred to by author, who noticed that paper only when his own was in press. Merit of author's paper lies in presenting an independent analysis of problem and, more importantly, in directing attention towards derivation of approximate solutions which should be of practical use in cases where an accurate solution would be excessively time-consuming.

H. C. Hopkins, England

2214. K. H. Swainger, Severe deformations, Proc. seventh int. Congr. appl. Mech. 1, 49–60 (1948).

Paper deals with some theories which lead to linear equations for analysis of some cases of finite displacements and strains with elastic, plastic, and flow components in anisotropic bodies, and when applied normal stress introduces slide strains. Stress and strain states of a material under complex loading are expressed in terms of normal strains in direction of, and transverse to, appropriate normal stresses. Forces and displacements are physical effects studied explicitly and these lead mathematically to "true" stresses and strains. The coordinates, lengths, areas, and volumes for current point are in the deformed body. Displacements and strains are of any finite severity attained incrementally. Superposition of loading is considered by following stress-"partial" strain history for each direction about current point. In a second loading the extended Huber-Mises-Hencky yield criterion gives orthogonal stress components which correspond to "equivalent" normal stress or octahedral shear stress attained by current point under first load. "Flow" strains are analyzed with assistance of a coefficient of viscous extension. "Slide" strains due to applied normal stress are analyzed with "induced" stress systems operating on an eight-face auxiliary element.

From author's summary by Bruno A. Boley, USA

2215. L. M. Milne-Thomson, Finite elastic deformations, Proc. seventh int. Congr. appl. Mech. 1, 33–40 (1948).

Paper presents in dyadic notation the results of F. D. Murnaghan [Am. J. Math. 59, 235–260 (1937)], the translation being more nearly literal than that given earlier by Murnaghan himself [von Kármán Anniv. Vol. 1941, 121–136].

C. A. Truesdell, USA

2216. Guiseppe Peretti, Significance of the arbitrary tensor which enters in the general integral of the equations of statics of continua (in Italian), Atti Sem. Mat. Fis. Univ. Modena 3, 77–82 (1949).

General solution of the equilibrium equation $t^{ij}{}_{,j} = 0$ for a continuous medium subject to no extraneous force was given by Finzi [Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (6) 19, 578–584 (1934)] in the form $t^{ij} = \epsilon^{ihr}\epsilon^{jks}\chi_{rs,hk}$, where χ_{rs} is an arbitrary symmetric tensor determinable only up to $u_{r,s} + u_{s,r}$ where u_r is an arbitrary vector. By suitable choice of u_r this result can be reduced to either of the classical solutions of Maxwell or Morera [Love, A treatise on the mathematical theory of elasticity, 4th ed., Cambridge University Press (1927), § 56]. In analogy to results which author attributes to Trefftz and Sobrero concerning Airy's stress function, he seeks a mechanical significance for the tensor $T_{ij} + \chi_{ij} + u_{i,j} + u_{j,i}$. He shows that it is possible to choose u_i in such a way that line integral $\int T_{ij}dX^j$ taken around a closed circuit equals moment of stresses acting upon inclosed area taken with respect to any origin on a certain line parallel to resultant force of stresses acting upon area.

C. A. Truesdell, USA

2217. M. J. Lighthill and F. J. Bradshaw, Thermal stress in turbine blades, Phil. Mag. 40, 770–780 (1949).

Problem of thermal stresses induced in a turbine blade by heating or cooling is investigated. Authors solve the thermoelastic equations for infinite slab which, initially, is at constant temperature, and suddenly has its surfaces in contact with a gas at given temperature. Then they determine thermal stresses in infinite wedge as a problem in plane strain, assuming an infinite heat-transfer coefficient. Approximate solutions for small wedge angle are finally obtained. They conclude that solutions at edge of wedge are bounded, and further, that there is a determinable

correspondence between stresses in slab and stresses at a point in wedge for which slab thickness is equal to wedge thickness at that point. They assume that the same conclusion holds approximately for a thin blade instead of a wedge. It is admitted that a principal obstacle to use of method is ignorance of heat-transfer coefficient. Test results show fair agreement with theory in predicting failure stresses for cooling, but errors in excess of 100% in estimates of temperature of gas to cause failure in heating.
William H. Hoppmann, II, USA

2218. **A. J. Moe, Secondary stresses; definition and relation to the safety factor** (in Danish), Bygnstat. Medd. 18, no. 3, 31–42 (1947).

2219. **H. Kauderer, On a nonlinear elasticity law** (in German), Ingen.-Arch. 17, 450–480 (1949).

It is pointed out that in those experiments where Hooke's law does not hold good, there arises need for studying nonlinear elastic deformation. This is done by assuming that strain components remain small, as in the classical theory, and that stress-deviation tensor is some nonlinear suitable function of strain-deviation tensor. Exact solutions are obtained for a simple tensor, shear and torsion of a circular cylinder, and results are compared with those given by experiment. Consequences of assuming other stress-strain laws are also worked out.

There is no reference to original work of W. Voight [Ann. phys. Chem. (Wiedemann) **7**, p. 536 (1894)], or to recent work of E. Sternberg [J. appl. Mech. 13, A53-A60 (1946)], in both of which small classical strains are used. No mention is made of the fact that nonlinearity which arises out of using finite strain components is at least as important as that which results in taking terms of an order higher than second in strain energy function [B. R. Seth, Third symposium on applied mathematics, Amer. math. Soc., McGraw-Hill (1950;) F. D. Murnaghan, Amer. J. Math. 59, 235–260 (1937)].
B. R. Seth, USA

Experimental Stress Analysis

(See also Revs. 2209, 2309, 2314)

2220. **H. T. Jessop and F. C. Harris, Photoelasticity: principles and methods,** New York, Dover Publications, Inc., 1950, viii + 184 pp., 164 photographs and diagrams. Cloth, 6 × 9 in., $3.50.

Book contains a brief introduction to photoelastic method of stress exploration. Authors state, in introduction, that their aims are to fill the need for a book to be read by an engineer with no extensive mathematical knowledge, and the need for an account of materials and methods now in use. They succeed in their first aim, by dispensing information in small tablets without attempting to build a broad understanding by logical development from a minimum number of basic laws. Sufficient information is given to enable reader to perform routine two-dimensional investigations.

Authors do not succeed as well in their second aim. Account of materials is sketchy and incomplete. A few of many modern methods in two-dimensional photoelasticity are described, but sections on three-dimensional theory and methods are limited to a few aspects of frozen-stress method. Conclusions regarding limitations of frozen-stress method, given at end of chapter on stresses in three dimensions, are not to be taken seriously in light of developments by Hiltscher and Drucker during the past decade. The scattered-light method, introduced by Weller in 1939, is not mentioned. Book contains a better account of geometrical optics of standard polariscopes than is to be found in other books on photoelasticity.

No references are given to technical literature, and the reader is referred only to the great treatise by Coker and Filon which does not, of course, contain the extensive developments of the past 20 years.
R. D. Mindlin, USA

2221. **J. A. H. Paffett, The application of photoelastic methods to ship design problems,** Trans. N.E. Coast Instn. Engrs. Shipb. 66, part I, 31–50 (1949/1950).

In first part of paper author reviews fundamentals of photoelasticity materials and techniques. He mentions an interesting method of compensating birefringence by turning an unloaded piece of Columbia Resin 39 which, due to manufacturing process has locked-up stress, about an axis parallel with the direction of minimum principal stress. He does not seem to be aware, however, of Tardy's method of rotating the analyzer, which is simpler.

To determine isoclinics, author designed a very compact portable polariscope. Light does not reach model perpendicularly, but at a small angle which author claims introduces only negligible error. To separate principal stresses as given by photoelasticity, author uses wire-resistance strain gages whose behavior on Xylonite he studied previously. Conclusion is reached that wire-resistance strain gages on Xylonite allow measurements with an error of plus or minus 10%, and that it would be reasonable to take a gage factor of 90% of the maker's figures.

Reviewer suggests that, to obtain stress trajectories in some of author's specimens, brittle-coating techniques are much more practical than the point by point determination of isoclinics with a portable polariscope.
A. J. Durelli, USA

2222. **C. E. Crede and F. Lambert, A shock recorder—comments on its construction and calibration,** Soc. exp. Stress Anal. 7, 103–118 (1950).

2223. **John J. Rebeske, Jr., Investigation of an NACA high-speed strain-gage torquemeter,** Nat. adv. Comm. Aero. tech. Note 2003, 41 pp. (Jan. 1950).

Development and performance of a strain-gage torquemeter are described. Report indicates instrument is capable of continuous operation over considerable periods of time. Performance is studied over a range of 5000 to 17,000 rpm with torque from 250 to 5500 in. lb. Effects of axial stress and slip-ring contact resistance are also presented. Calibration constants are shown to be stable. Instrument is reported to have a precision of ±0.33% of full scale.
C. Dohrenwend, USA

2224. **Claude M. Hathaway, New developments in dynamic strain recording equipment,** Soc. exp. Stress Anal. 7, 119–126 (1950).

2225. **Carl G. Hard af Segerstad, A new pneumatic method for measuring small variations in length, and some applications of it,** Tekn. Forsk. 20, 107–119 (Aug. 1949).

Author describes a group of instruments for measurement and control of very fine tolerances based on what is called delta principle. Essentially, such an instrument consists of a spindle carrying three disks which is free to move in an axial direction within a cylindrical chamber. Two of the disks act respectively as inlet and outlet to a measuring chamber supplied with air from a source at constant pressure. A movement of spindle changes sizes of two orifices and so produces a change in pressure in chamber which can be registered on a differential manometer. Over a wide range, manometer pressure can be made to vary linearly with impulse applied to spindle. All instruments described combine simplicity of construction with exceptionally high sensitivity.
Maurice Holt, England

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 2152, 2191, 2204, 2267, 2272, 2278, 2294, 2323)

2226. Nikolaj Panišin, Analysis of elastically supported beams by deformation method (in Czech), Technický Obzor SIA, 58, no. 1, 7–10 (Jan. 1950).

More accurate method demonstrated on examples is compared with approximate method developed by Prof. R. Saliger [Reinforced Concrete, Leipzig (1931) and also El Hormigón Armato, Barcelona (1943)]. For a system of crossing beams it is found that values of bending moments only in central part check fairly with author's computation, whereas bendings moments in beams approaching rigid supports calculated by Sliger's method are up to 42% smaller than their exact values. Author's calculations are based upon his previous paper published in Technický Obzor no. 5 (1945). J. J. Polivka, USA

2227. Ernst Melan, An approximation for the calculation of support moments of continuous beams (in German), Schweiz. Bauztg. 67, 409–411 (July 1949).

Method of analysis of continuous beams developed in paper is similar in derivation and procedure to that of Hardy Cross in that it permits solving for moments at supports to any desired degree of accuracy by progressively converging corrections, convergence being same as for the Cross method; but it requires only one half the number of multiplications and additions, and hence is much faster. An example is given showing necessary operations in a tabular form, somewhat cumbersome for clarity, but procedure introduced by Cross of noting calculations below a sketch of beam could be equally applied to this method.

A. R. C. Markl, USA

2228. Hisao Takeyama, On the bending strength and vibrations of radially rotating bars. II, III (in Japanese, with English summary), J. Soc. appl. Mech. Japan 2, 91–96, 110 (1949).

[For part I, see Rev 2, 980.] Deflection of a rotating bar like a turbine blade, acted on by a single load, is obtained analytically; formula is suitable for numerical and graphical integrations. When bar is hinged at end near rotating axis, its deflection caused by a concentrated load is found to have singularities at zero angular velocity. It is made clear that these singularities are apparent and have no effect on solutions of integral equations of vibrations. From author's summary

2229. F. Stüssi, Funicular polygons and differential equations, Bull. Soc. roy. Sci. Liége 18, 272–284 (1949).

Author develops a method of obtaining a numerical solution of the differential equation $y'' + cy + F = 0$ for a finite interval, where c is a constant and F is a given function of x. Interval is divided into n equal parts. Continuous function y is then replaced by a set of values y_m ($m = 0, 1, 2, \ldots, n$) obtained as follows. Function y is regarded as the load acting on a simply supported beam. Beam is then divided into n equal portions, each simply supported, and load on each portion is assumed to vary linearly or quadratically. Values y_m are then defined to be loads acting on each of the $n + 1$ supports. Other functions in above differential equation are treated similarly. By use of theory of funicular polygon, the set of values of y'' are expressed in terms of the set y_m. Above differential equation is then reduced to a set of algebraic linear equations for y_m. As an example, solution is obtained when F is a constant, with $n = 4$. In this case, it is found that y_2 differs from its exact value by 0.13%. It is noted that when same problem is solved by simple finite differences, y_2 has an error of 2.7%. Case when differential equation contains y' can be treated similarly. It is stated that method also applies when c is a given function of x.

Courtesy Mathematical Reviews G. E. Hay, USA

2230. Aris Phillips, On symmetrical pure bending of beams, Stanford Res. tech. Rep. 4, 27 pp. + 10 plates (Aug. 1949).

Object of paper is to present a comparatively simple analysis of bending in range of plastic flow for use in design. Stress-strain relation is approximated by an elastic line and a linear work-hardening line, which is same in tension and compression. Usual theory is presented in form which gives a rapid approximate relation between maximum strain and bending moment. Sections with two axes of symmetry are treated, and also those sections with one axis of symmetry for which neutral axis remains approximately fixed in section. E. H. Lee, USA

2231. W. Swida, On the deformation of beams in the elastoplastic state (in German), Ingen.-Arch. 17, 71–87 (1949).

It is known that deflection of an elastic beam can be obtained as the bending moment of the conjugate beam under a distributed load which is given by reduced bending moment of original beam (i.e., bending moment divided by bending stiffness). Author points out that, for statically determinate beams at least, this method of determining deflection is readily extended to elastoplastic beams by defining reduced bending moment in a suitable manner. Relation between actual bending moment and reduced moment is discussed for various structural sections, and elastoplastic deflections of simply supported beams and cantilever beams under various types of loading are determined.

W. Prager, USA

2232. I. G. Hannemann, *v*-forces and *w*-moments (in Danish), Bygnstat. Medd. 19, no. 4, 107–121 (1948).

The v-forces and w-moments are fictional forces and moments such that, when applied as loading on the conjugate beam, they produce a moment curve identical with the deflection line of given beam. Author deduces formulas for v-forces and w-moments and proves that any deflection line may be presented by moment curve resulting from loading the conjugate beam by such quantities.

From author's summary by A. R. Holm, Denmark

2233. Sven G. Bergström, An approximate method for calculating additional moments in members under action of combined compressive and bending forces (in Swedish), Inst. Brobyggnad Medd. no. 3, 40–52 (1948).

Author reviews methods for calculating beams under simultaneous action of lateral load and axial forces. He suggests a new approximative method consisting of representing form of elastic line by a sum of "buckling functions." (Buckling functions define the elastic lines as they arise during buckling of beam.) Author states formula for additional moment. He points out that, in general, buckling functions are not orthogonal. Using different examples, author obtains very useful approximations by means of his method. In a discussion, Ödman points out that good convergence of series depends on choice of first member of series. The greater the deviation of first member of series from final expression, the greater will be error in moment. Ödman prefers use of eigenfunctions to that of buckling functions.

Ludwig Föppl, Germany

2234. W. Swida, The elastoplastic bending of a curved bar of work-hardening material (in German), Ingen.-Arch. 17, no. 4, 343–352 (1949).

Author obtains relation between bending moment and fiber

stress in a curved beam symmetric with respect to plane of bending upon assumption of the same stress-strain relations in tension and compression. These relations are represented by a straight line through origin with slope E equal to Young's modulus and two parallel straight lines with slope $E_1 < E$ which intersect axis of stress at $\pm\sigma_a$. Author further assumes that cross section is of such shape that under increasing moment, inelastic action commences at concave surface of beam. Slopes and deflections are considered briefly. Rectangular cross section is treated in detail, and numerical examples are worked out.

Finally, there are remarks on economy to be realized in working in inelastic range, and solution of problem is indicated for curved stress-strain diagrams. Wm. R. Osgood, USA

2235. **Th. Wyss, Bending stresses in bearer cables of cable railways in neighborhood of the load** (in German), Schweiz. Bauztg. 67, no. 38, 525–531 (1949).

Author considers bending stresses in cables under tension, acted upon by a transverse concentrated load. Types of cables considered are: solid wire, spiral wire cable, and covered wire cable. Isaachsen [(EMPA, Switzerland), report 166] has given formulas for these bending stresses both when friction between wires in a cable and when slipping of wires in a cable are negligible. Paper reports on experimental tests for validity of these formulas.

In case of solid wire there is good agreement between theory and experiment. In case of other two cables, experimental values lie between theoretical values corresponding to negligible friction and negligible slipping, and are obtained approximately, for practical purposes, if the theoretical values for negligible slipping are multiplied by 0.6. G. E. Hay, USA

2236. **J. A. Haringx, On highly compressible helical springs and rubber rods, and their application for vibration-free mountings, IV,** Philips Res. Rep. 4, 261–290 (Aug. 1949).

Paper, fourth of a series of six, is primarily concerned with an analysis of so-called "vibration-free" mountings, particularly for sensitive instruments. Author compares (1) a simple resilient mounting without damping, (2) a resilient mounting with vibration absorber (auxiliary mass), damping being provided between absorber and main mass, and (3) simple resilient mounting with damping between mass and foundation. It is shown that for steady state vibration of foundation (3) is of advantage for frequencies near resonance as compared to (1), but is disadvantageous for frequencies higher than $\sqrt{2}$ times resonant frequency. By using vibration absorber (2) with an auxiliary mass approximately equal to main mass, disadvantage may be overcome while at same time retaining advantage for frequencies near resonance. Further study is made on basis of decay of free vibration for various systems, object being to provide a design in which free vibration will die out as quickly as possible when mass is subject to an initial impulse or displacement. In comparing systems (2) and (3), optimum values of mass ratio and stiffness parameter are chosen, while damping parameters are so selected that equal times are required for the two systems to reach 0.01 amplitude ratio during free vibration after an initial impulse. Under these conditions author concludes that system (2) with vibration absorber is superior to damped resilient mounting (3) with respect to forced vibrations at high frequency ratios, and is also less sensitive to incidental changes in damping coefficient.

A. M. Wahl, USA

2237. **J. A. Haringx, On highly compressible helical springs and rubber rods, and their application for vibration-free mountings, VI,** Philips Res. Rep. 4, 407–448 (1949).

Paper concludes a series of six articles on above subject, and treats practical design of so-called vibration-free mountings for instruments, based on results and formulas developed in preceding articles. Design of a "universal" mounting which is generally satisfactory with respect to disturbing vibrations occurring in any direction is considered for two cases: (1) a mass with a damped vibration absorber and supported by helical springs, and (2) a mass supported by rubber columns, damping being obtained by virtue of inherent damping in rubber. In both cases, various natural frequencies of main mass are taken equal. Particular emphasis is placed on design of helical springs (case 1) so as to obtain proper lateral and axial rigidities; charts are presented to facilitate the work. To obtain damping between main mass (or mounting) and absorber, a viscous damper consisting of two cups of square cross section and having a viscous liquid in space between cups is used; equations are worked out for this case. Results are applied to case of a mounting of 100 kg weight with absorber. Details are also worked out for a simple mounting of 900 kg weight without absorber (case 2) supported by four flexible rubber columns, one at each corner. Advantages and disadvantages of the two designs are discussed from standpoint of use for instrument mountings. A. M. Wahl, USA

2238. **R. B. Heywood, Tensile fillet stresses in loaded projections,** Proc. Inst. mech. Engrs. 159, no. 45, 384–398 (1948).

Photoelastic method was employed to measure stresses in fillets of single or multiple projections (which consisted of short, flat, cantilever beams that had straight upper and lower faces with crests and fillets of constant curvature). Over-all length of projection, pitch, radius of fillet, and flank angle were varied; loading of a single or of two adjacent projections was carried out by means of special loading fixtures. Loading an adjacent projection was found to produce only negligible reduction in maximum stress in tensile fillet, but did produce marked change in maximum stress in compressive fillet. By supplementing data with results of Dolan and Broghamer [Univ. of Ill. Engng. Expt. Sta. Bull. 335 (1942)] a long and rather cumbersome empirical formula was developed for maximum fillet stress in terms of load, thickness, and five dimensions of projection. Author suggests this formula is applicable to a wider variety of conditions, ranging from a long cantilever to a stub-shaped projection loaded at positions close to fillet; formula gives results with considerably greater accuracy than can be obtained by using a modified Lewis formula. Author extends formula to the determination of fillet stresses in various types of screw-thread forms, and concludes that Whitworth thread form is remarkably close to optimum design. Data are also intended for application to determination of maximum fillet stresses in other types of loaded projections, such as gear tooth.

Thomas J. Dolan, USA

2239. **Rudolph Liebold, The bending of a leaf spring rigidly clamped at both ends** (in German), Z. angew. Math. Mech. 28, 247–249 (1948).

Handbook formulas for deflection of beams fixed at both ends, as author points out, are based on assumption of no restraint in direction of beam axis. However, in many cases this assumption is incorrect, and a tensile force will be induced in beam by elongation of its center line due to bending. In such cases, load will not be proportional to deflection and stress. Author then proceeds with mathematical analysis of problem of a leaf spring of length $2L$ fixed at both ends, and subjected to a central transverse force P.

The solution gives maximum deflection as:

$$f = (PL^3/4EI)[(\lambda L/2 - \tanh \lambda L/2)/(\lambda L/2)^3]$$

where E is Young's modulus, I second moment of area of section,

$\lambda = (S/EI)^{1/4}$ and S is the tensile force due to restraint. For $S = 0$, above expression is identical with formulas given in handbooks.

Tensile force S can be calculated from elongation of center line due to bending. The quantities P, f, fixing moment, and maximum stress (σ_{max}) can be expressed as functions of λL. Author works out a numerical example plotting f and σ_{max} as a function of P. Graphs show the nonlinear characteristics, and indicate that both deflection and stress are below values obtained from conventional formulas. Nicholas Sag, Australia

2240. H. Nylander, Method to increase rigidity in torsion of double-flanged beams (in German), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 327–332 (1948).

Author studies influence of lateral stiffeners upon torsional rigidity and torsional stress of I-beams. Beam is twisted by end moments causing not only torsion but a combination of torsion and flexure of flanges due to stiffeners. Stiffeners reduce warping of cross sections of beam. Timoshenko solved the influence of nonwarping cross sections at ends of an I-beam and at center of a rectangular beam [*Theory of elastic stability* (1936), p. 256, and *Theory of elasticity* (1924), p. 274].

Author furnishes results of influence of partially warped cross sections at any number of positions along longitudinal direction of I-beams. Flexural rigidity of flanges of I-beam is evaluated as a function of torsional rigidity of stiffeners.

An influence of the combination of torsion and flexure by several stiffeners is expressed by substituting a different coefficient for torsional rigidity coefficient in torsional moment equation. Derivation of this coefficient is omitted but reference is made to a previous article by author [Diss. Stockholm (1942)]. Application of result of this investigation is made to an I-beam with two restraining lateral stiffeners. It is claimed that torsional stress at longitudinal center is more than halved by stiffeners. No independent references or test data are furnished with which to compare or check this remarkable result. F. B. Schneider, USA

2241. H. de Leiris and J. Barthélemy, Deformation and stresses in plane curved tubes (in French), Proc. seventh int. Congr. app. Mech. 1, 308–321 (1948).

Authors present a brief progress report on theoretical and experimental results obtained by French investigators on title subject. Two papers [see REVS 3: **1060, 1061**], in particular, contain the essential results. W. H. Hoppmann, II, USA

Plates, Disks, Shells, Membranes

(See also Revs. 2152, 2186, 2210, 2211, 2213, 2261, 2264, 2289, 2290, 2295, 2329)

2242. Ya. S. Uflyand, Integral equation for the bending of a sectorial plate with clamped radii (in Russian), Doklady Akad. Nauk SSSR 72, 251–254 (1950).

Author considers a thin horizontal plate, thickness h, in form of a circular sector, radius R, central angle 2γ, clamped on the radial edges, and loaded vertically by a distributed load q. Problem is to determine deflections of plate and maximum moment on one of clamped edges. The series solution of the differential equation governing deflection of plate leads to two integral equations which reduce to one if load q is symmetrical with respect to axis of symmetry of plate. Author discusses only this last case but includes also the situation when edge on arc of sector is clamped; this gives very simple boundary conditions. Method is as for a plate simply supported on edges; hypothetical distributed loads are applied along radial edges in such a way that deflections and their first derivatives at these edges equal zero.

Resulting integral equation is solved approximately by substituting concentrated forces for the hypothetical distributed loads along radial edges. At the end numerical values are tabulated for deflection at center of plate and maximum moment on clamped arc for following conditions of loading: (1) uniformly distributed load q over the whole area of the plate; (2) concentrated load P at the center of the plate; (3) uniformly distributed load p along the central arc of the plate, $r = R/2$. All three cases are evaluated for $\gamma = \pi/8, \pi/4, \pi/2$. Deflections and moments are given in a dimensionless form explained in paper.
Courtesy Mathematical Reviews. T. Leser, USA

2243. Carlo Tolotti and Giuseppe Grioli, On the calculation of plates with ribs (in Italian), G. Gen. civ. 87, 275–308 (June 1949).

Methods of calculation of plates with ribs have been given by different authors but, because of arbitrary assumptions about stress distribution, have not been considered satisfactory. Paper proposes a new method based on equality of deformation of plate and of ribs. Two cases are considered: a simply supported plate, and a plate fastened to the girders. In second case two approximations are contemplated: (1) Only vertical deflections are investigated; and (2) twist of girders is taken into account. Displacements in plane of plate, due to deflection, are neglected. Solution is reduced to integrodifferential equations. By a development in series, they are reduced to a system of linear equations, the number of which depends only on number of terms of series, i.e., of desired approximation.

Applications are given to a rectangular plate with simple diagonal ribs, and to one with a double row of ribs inclined at 45 deg to edges. Numerical examples are given. Publication of a manual with a complete discussion and numerical tables is contemplated.
Aurel A. Beleş, Rumania

2244. Piero Pozzati, Rectangular slab, elastically supported on its boundaries, under any loading condition (in Italian), G. Gen. civ. 87: no. 1–2, 55–67; no. 5, 232–243; no. 7–8, 361–371 (1949).

Essential tool used in paper is solution for a rectangular plate simply supported along two opposite edges and loaded on other two by vertical forces distributed according to a sine law. The reaction at supported edges is developed into a Fourier series. Such solutions are superposed to solution for a plate simply supported at all edges, and the coefficients of this superposition are so determined that reaction becomes zero everywhere except at corners, or so that deflection of edge matches with that of a beam supporting this edge and receiving forces from plate as a bending load. For this purpose, an infinite set of linear equations must be solved. Examples given in paper show that convergence is good enough to make this an easy task. By adding some concentrated forces pointing upward to load on plate, it is possible to even make the reactions at corners vanish and to obtain solutions for mushroom-like plates, resting only on one or two supports at interior points. W. Flügge, USA

2245. H. D. Conway, Bending of rectangular plates subjected to a uniformly distributed lateral load and to tensile or compressive forces in the plane of the plate, J. appl. Mech. 16, 301–305 (Sept. 1949).

Paper presents a method of determining deflection and stresses in a simply supported rectangular plate subjected simultaneously to a uniform lateral load and to uniform tensile or compressive forces in plane of plate. A solution for the deflection is determined in the form $w = \sum_1^\infty Y_m \sin m\pi x/a$. Method gives an equation that requires a single summation, but resulting equation is much

more complicated than that resulting from Navier method. Graphs are given whereby maximum stress and deflection may be easily calculated. R. M. Wingren, USA

2246. **T. E. Schunck, The square plate under shear load beyond the buckling limit** (in German), Ingen.-Arch. 17, nos. 1 & 2, 119–128 (1949).

Author shows that in many problems concerning rectangular plates, because of mathematical difficulties, boundary conditions on small sides are neglected and plate is assumed to be a simple strip supported on two sides only. Under such conditions, he shows that Kromm and Marguerre, in Germany, studied a rectangular plate loaded in its plane by normal and shearing stresses beyond the buckling limit. Paper establishes difference between this approximation and plate supported on all four sides. Author chooses a square plate—where the differences are more evident—and by means of finite differences and by an iteration method gives deformation and stresses for a clamped plate with rigid edges loaded by shear stresses beyond buckling limit. Conclusion is reached that differences are small even for square plate, which is stiffer than the strip. Aurel A. Beleş, Rumania

2247. **J. Hlitčijev, On longitudinal ribs in a compressed plate. With an appendix by J. Karamata** (in Serbian), Glas Srpske Akad. Nauka 195, 17–36 (1949).

Problem of a plate reinforced by longitudinal ribs treated by Timoshenko [Theory of elastic stability, McGraw-Hill, New York-London (1936), pp. 372 ff.]. Numerical results were given only for the case of two or three ribs. Author shows that problem of optimal dimensions can be treated for an arbitrary number of ribs using closed analytic expressions which Karamata obtained for the series $\sum_{-\infty}^{+\infty} \{\lambda - (1 + (2sn + t)^2)^2\}^{-1}$ in terms of trigonometric and hyperbolic functions.

Courtesy Mathematical Reviews W. Feller, USA

2248. **L. S. D. Morley, Load distribution and relative stiffness parameters for a reinforced flat plate containing a rectangular cut-out under plane loading,** Nat. Luchtlab. Amsterdam Rep. S.347, 12 pp. (1949).

A method is given, based on conventional shell theory, for determining load distribution and relative stiffness parameters for a three-bay flat monocoque plate containing a rectangular cutout in center bay and under plane loading. Paper is an introduction to a general theory for distribution of loads throughout open and closed shells containing cutouts.

From author's summary by Paul Naghdi, USA

2249. **L. E. Grinter, Plane state of stress studied by use of a physical grid,** Proc. seventh int. Congr. appl. Mech. 1, 139–150 (1948).

A method is outlined for analysis of solid plates under plane stress by means of physical-grid analogy. The task is reduced to solving analogous grid by means of relaxation and distribution. In spite of its straightforwardness, method is too tedious to be applied in design office. It may be looked upon as a research tool which needs further simplification and more effective means for acceleration of convergence. Procedure has been used by author to solve following problems: plate stresses due to weld shrinkage; stresses due to rectangular notches in thin beams; and gusset-plate analysis. M. Gololobov, Czechoslovakia

2250. **W. Zerna, Contribution to the general theory of bending of shells** (in German), Ingen.-Arch. 17, nos. 1 & 2, 149–164 (1949).

Vector and tensor methods combined with some results of differential geometry of surfaces are used to discuss bending of shells with special reference to elastic shells. Author is apparently unaware of similar contributions by Russian authors, whose work is reviewed by A. L. Goldenveizer and A. I. Lurye [Prik. Mat. Mekh. 11, 565–592 (1947), English translation no. 9, Amer. Math. Soc. (1950)]. B. R. Seth, USA

2251. **F. Martin, The spherical membrane shell under concentrated load** (in German), Ingen.-Arch. 17, no. 3, 167–186 (1949).

Mercator's projection is a conformal mapping of a sphere on the x-y-plane. Real and imaginary parts of any function of complex variable $x + iy$ are connected with a system of membrane forces in a spherical shell. Loads producing these forces correspond to singularities of generating function. In this way simple solutions are found for a sphere loaded at an arbitrary point by a tangential or radial force and supported at poles. Inextensional deformations are treated similarly. In order to find displacements for a given system of membrane forces, a plane-potential equation must be solved. Some generalizations for other shells of revolution are indicated. W. Flügge, USA

2252. **A. E. Sneddon, The strength of thin concrete walls in axial compression under distributed loading,** Publ. int. Assn. Bridge Struct. Engng. Final Rep., 589–596 (Sept. 1948).

Test results are reported for strength of 4-in. concrete walls of single-story height under distributed axial compression. Results for various wall lengths are related to strength of certain control specimens. M. Goland, USA

2253. **William Thomas Marshall, A method of determining the secondary stresses in cylindrical shell roofs,** J. Instn. civ. Engrs. 33, no. 2, 126–140 (Dec. 1949).

The membrane theory for stresses in thin shells is based on the fundamental assumption that there is no change of shape of shell on loading. In shell-roof construction, however, there must be a deflection of the edge tie-beam, with resultant secondary stresses in shell. Some solutions have been given for this problem (Dischinger, Finsterwalder, Schorer and Jenkins), but they are often very difficult to follow, especially by engineers, because of lack of numerical examples. Author has utilized the property shown by Dischinger—that longitudinal stress-distribution in the cylindrical portion of shell is almost equal to that obtained by considering the complete shell as a beam spanning between the columns at the gable ends—in order to determine the constants of integration in Schorer's method of analysis. This "beam" method is suitable for numerical solution of problems, as is shown at end of paper in the calculation of an example involving design of a roof.

From author's summary, by M. Hampl, Czechoslovakia

2254. **Wei Zang Chien, Derivation of the equations of equilibrium of an elastic shell from the general theory of elasticity,** Sci. Rep. Nat. Tsing Hua Univ. Ser. A. 5, 240–251 (1948).

Macroscopic equations of equilibrium of an elastic shell are derived from three-dimensional theory of elasticity by suitable integrations across thickness of shell. Tensor notation is employed. E. Reissner, USA

2255. **W. F. Brown, Jr. and F. C. Thompson, Strength and failure characteristics of metal membranes in circular bulging,** Trans. Amer. Soc. mech. Engrs. 71, 575–585 (1949).

Paper presents tests of several disks subjected to hydrostatic pressure. Considerable care was apparently given to plotting strains and pressures. However, one disappointing fact is that

authors set up general expressions for effective stress and effective strain but do not plot these values. Paper presents a very ingenious way of determining critical strain until membrane becomes unstable. R. G. Sturm, USA

2256. H. C. Engel and T. P. Pajak, Honeycomb-sandwich structures, Prod. Engng. 20: I, no. 8, 81–85; II, no. 10, 131–134 (1949).

A brief review of experiences with honeycomb-sandwich structures at the Glenn L. Martin Co. Part I deals with properties of various types of honeycomb cores, materials used as facings, FM-45 adhesive and strength properties of sandwich beams and panels. In part II, accelerated weather tests, heat-insulation and sound-transmission properties and detail design factors (edging, inserts, attachments and splices, repairs) are discussed. Applications in Martin 202 are indicated. F. J. Plantema, Holland

Buckling Problems

(See also Revs. 2233, 2240, 2247, 2255, 2290, 2291)

2257. E. I. Grigolyuk, Approximate solution of a problem on the stability of a ring under torsion (in Russian), Prikl. Mat. Mekh. 14, 99–101 (1950).

Author considers a thin circular plate of radius b with concentric hole of radius a, thickness of the plate δ, and torsional rigidity D, fixed on the outer and inner boundaries and turned by a moment M applied to the boundaries in the horizontal plane. Problem is to find the critical value of this turning moment for a given ratio of the radii a/b. Exact solution of the differential equation governing deformation of a ring is inconvenient for numerical computations. Author presents it but uses it only for comparison with results obtained from the approximate solution which he calls Galerkin solution. At the end he tabulates critical values of the moment in dimensionless form, $k = M/\pi D$, for 6 ratios a/b. Two of those values are compared with the ones obtained from exact solution and they differ by 12 and 7.2% respectively.

Courtesy Mathematical Reviews T. Leser, USA

2258. L. A. Ringelstetter, Buckling tests of flat plywood plates in compression with face grain 45° to load—loaded edges clamped, others simply supported, For. Prod. Lab. Rep. 1316-J, 7 pp. (1949).

Formula $P_{cr} = k_c E_L h^2/a^2$ is used to determine experimentally critical buckling load P_{cr} for a flat plywood plate with grain direction at an angle of 45 deg to edges. k_c is a constant, E_L Young's modulus in direction parallel to grain, h thickness of plate, and a length of clamped loaded edges. Unloaded edges are simply supported. Results are compared with those for similar plates with all edges simply supported. No comparison has been made with any mathematical results for such plates. These can be obtained by using strain energy method, as has been done by S. G. Lekhnitskiĭ [Anistropic plates, Moscow, pp. 264–268 (1947)].

B. R. Seth, USA

2259. G. Schnadel, Exceeding the buckling limit in thin plates, David Taylor Model Basin Transl. no. 64, 11 pp. (Dec. 1949).

Paper, which was published in 1930 in Proceedings of 3rd International Congress of Applied Mechanics, has been recently translated into English by M. C. Roemer. Method is outlined for an approximate buckling solution of nonlinear differential equations of bending of a plate assuming large deflections. Solution is expressed as a sum of two solutions. First represents a solution for constant stresses σ_x and σ_y, and the second the necessary correction to determine the nonuniform stress distribution. Latter is obtained by minimum energy considerations. A scheme for handling rectangular plates with joints is also outlined.

R. L. Bisplinghoff, USA

2260. A. Manidakis, Incongruity in Euler's buckling theory and the derivation of a formula applicable to all degrees of buckling (in Greek), Techn. Chron. 26, 305, 579–584 (1949).

Euler assumes that modulus of elasticity E is constant for all degrees of buckling and disregards direct compressive stress induced by column load P in every cross section of a column of length l and cross-sectional area a. He thus overlooks effect of this stress on buckling load and on the value of E.

Author considers E to be variable and given by $E = d\sigma/d\epsilon$, corresponding to a $\sigma = P/a$, which may be obtained from compression stress-strain curve of the column material. Using above values of E and σ, following expressions for critical buckling length, breaking length, and maximum moment acting in column are derived, respectively:

$$l_c = \pi K(E/\sigma)^{1/2};\ l_b = 2\pi K(E/\sigma)^{1/2};\ M_{\max} = (2Pl/\pi) \times [\mu(1-\mu)^{1/2} - (1-\mu)]^{1/2}$$

where K is radius of gyration of cross-sectional area of the column, and $\mu = \pi(EK^2a/P)^{1/2}/l$.

Validity of above expressions is verified satisfactorily by experiment. Dimitri Kececioglu, USA

2261. G. Winter, Performance of compression plates as parts of structural members, Research: Engng. Struct. suppl. (Dec. 1949).

This is a summary review of recent investigations on behavior of compression plates as parts of structural members, in contrast to that of isolated plates. Basic theory of different types of problems is described, e.g., those in which elastic stress limits are exceeded, and those in which development of membrane stresses allows loads above buckling loads to be carried. Results of simplified approximate solutions are, in both cases, compared with those of more elaborate methods of calculation. Tests on lipped channel sections are briefly described, showing how the transverse deformations of compression flange determine effective cross-sectional properties. Some effects of buckling in thin-walled compression members, such as box columns, are mentioned. Bibliography lists 22 items. P. S. Symonds, USA

2262. C. F. Kollbrunner and G. Herrmann, Buckling of elastic plates in elastic range (in French), Tech. mod. Constr. 4, 375–377 (Dec. 1949).

Buckling of rectangular flat plates under end loading is analyzed by energy methods to determine buckling stress and the shape of buckled surface. End load is assumed to be linearly variable over width of plate; edges are assumed to be supported in various ways. Buckling stresses are expressed in terms of Euler stress, by means of a coefficient. Shape of surface after buckling is shown by means of contour lines. F. R. Shanley, USA

2263. Murry Schildcrout and Manuel Stein, Critical combinations of shear and direct axial stress for curved rectangular panels, Nat. adv. Comm. Aero. tech. Memo. 1928 (Aug. 1949), 29 pp.

Problem of buckling of a simply supported rectangular curved panel under combined shear and axial stress is solved by means of Galerkin method. Theoretical combinations of stress coefficients are tabulated for panels with different aspect ratios. Empirical

corrections are suggested for panels under combined shear and axial compression. Conrad C. Wan, USA

2264. **Paul Seide, Shear buckling of infinitely long simply-supported metalite type sandwich plates,** Nat. adv. Comm. Aero. tech. Note 1910, 20 pp. (July 1949).

A theoretical solution is obtained for critical shear stress in elastic range. Critical shear-stress coefficients are given graphically for a range of core stiffness factor from zero to unity. An approximate correction is also suggested for determination of critical shear stress in plastic range. Conrad C. Wan, USA

2265. **Georg Wästlund and Sven G. Bergström, Buckling of compressed steel members,** Trans. roy. Inst. Technol. Stockh., no. 30, 172 pp. (1949).

Monograph presents a clear review of much of the standard theory on buckling of bars, frames, and built-up columns as well as some new test data on buckling of bars and of simple frames. Purpose is to present development of a new Swedish specification on structural buckling. Proposals for design of batten plates and lattice bars are given in the latter portion. There is special consideration of influence of shear on buckling load. Eccentricity also is discussed. A bibliography of 55 references covering only continental European work in this field is provided.

Reviewer's opinion is that authors should have treated numerical methods for determining theoretical buckling loads. Also, use of experimental results published in England and in USA would have added to value of the monograph.

W. H. Hoppmann, II, USA

2266. **K. von Sanden and F. Tölke, On stability problems in thin cylindrical shells,** David Taylor Model Basin Transl. 33 (from German), 63 pp. (Dec. 1949).

Translation of paper appearing in *Ingenieur-Archiv* in 1932. After an introductory section on the elasticum, theoretical critical stresses of an unstiffened cylinder are worked out for axial compression, circumferential compression, hydrostatic pressure on a closed cylinder, and torsion. Buckling stress of a closed, ring-stiffened cylinder under hydrostatic pressure is also treated. So far as the unstiffened cylinder is concerned, the analysis used is substantially equivalent to analyses appearing in textbooks by Flügge and Timoshenko a few years after publication of the original paper. Principal interest in the work at the present time, therefore, is treatment of ring-stiffened cylinder buckling under hydrostatic pressure. S. B. Batdorf, USA

2267. **S. R. Sparkes, J. C. Chapman, and A. J. S. Pippard, Experiments on the flexure of rectangular box girders of thin steel plating,** Eng. Struct., 61–92 (Dec. 1949).

Paper described an extensive experimental investigation of rectangular box girders of thin steel plating subjected to pure flexure. Seven unstiffened girders of square section 8 in. × 8 in. × 60 in. long, as well as two stiffened girders 68 in. wide, 45 in. deep, and 150 in. long were tested. Each was subjected to four-point loading so as to produce pure flexure in its test length, which was approximately 50 per cent of its over-all length. Four of the small boxes had thin flanges and relatively thick webs, while three had thin webs but thick flanges. Thickness of steel plate from which the boxes were fabricated ran from 0.05 in. to 0.25 in. Deflections were measured with various dial gages, and strains were determined with numerous electrical resistance gages. Test results are given extensively in form of curves and deflection contours.

No theoretical analysis is given, although some remarks are made in conclusions about discrepancies between experimental results and certain theoretical calculations. Reviewer was disturbed by pessimistic viewpoint expressed in last sentence under conclusions which reads: "The authors consider that in general the problems of thin-walled structures are too complex to be amenable to adequate mathematical treatment,and there is need for more experimental data from which rational design rules may eventually be derived." W. H. Hoppmann, II, USA

Joints and Joining Methods

(See also Rev. 2324)

2268. **W. D. Mitchell and D. Rosenthal, Influence of the elastic constants on the partition of load between rivets,** Soc. exp. Stress Anal. 7, 17–26 (1950).

Description of extensometer used is typical of paper: so lacking in essential detail that any check of doubtful procedures, assumptions, or conclusions is impossible.

After substituting bolts for rivets, authors appear to have measured over-all displacement of their double-lap joint with one leg of extensometer on a part of joint almost certain to have been affected by local deformations. Data obtained appear then to have been analyzed by an electric analog using quantities of doubtful validity, since computed deformation-of-bolt factors from which pertinent resistances were determined were shown to be in poor agreement with the experimental.

Conclusions appear to be reasonable.

Joseph S. Newell, USA

2269. **N. D. Tarabasov, Strength calculation of pressed joints** (in Russian), Doklady Akad. Nauk SSSR 67, no. 4, 615–618 (1949).

Stresses are analyzed in an assembly consisting of m annular concentric rings shrunk into each other and into a central circular disk, and subject to uniform shear along outer edge caused by an applied torque. Mechanical properties of all components are assumed identical and elastic. Numerical results are given for one particular case consisting of two rings and the central disk. It is maintained that given solution is of particular importance for rotating assemblies of this type where, with due regard to inertia forces, it enables adjustment of initial radii (before shrinking) to obtain desired stress distribution. George Winter, USA

2270. **A. De Marneffe, Constitution of assembly joints, the best shape to give the gussets in frame systems and the importance of their influence on stresses of the structure as a whole** (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 277–284 (1948).

Structures

(See also Revs. 2191, 2195, 2207, 2226, 2232, 2243, 2249, 2252, 2253, 2256, 2264, 2531)

✎2271. **Arthur G. Hayden and Maurice Barron, The rigid-frame bridge,** 3rd ed., New York, John Wiley & Sons, Inc., 1950, x + 240 pp., 77 figs., 15 photos, tables & charts. Cloth, 6 × 9¼ in., $5.

When first published in 1931 this book treated what was essentially a new development in steel and reinforced-concrete bridges. Method of analysis presented is based upon use of Maxwell's equations and may be applied with or without influence lines. Main value of book, however, rests in its presentation of authors' experience in terms of design data. For example, a useful point is made regarding proper assumption of pin-end bases even though columns are poured integrally with footings. Contains a rather

complete study of skew type of rigid-frame bridge in simplified form for use in a design office. L. E. Grinter, USA

2272. E. C. Hartman, R. L. Moore, and F. E. Rebhun, Tests of aluminum and steel railway bridge girders, Bull. Amer. Rly. engng. Assoc. 51, 290–306 (Dec. 1949).

New railroad bridge over Grasse river at Massena, N. Y., consists of seven deck plate girder spans, one built entirely of aluminum, the remainder of steel. Tests on the 97.5-ft aluminum span and on a 98-ft steel span under static and moving loads permitted an interesting comparison between behavior of steel and aluminum girder bridges. Measured strains and deflections under static loads were in fair agreement with computed values, when additional stiffening effect of splices and lateral bracing system is taken into consideration. Under moving loads with speeds from 28 to 34 mph, primary stresses and vertical deflections in both bridges were only 12 to 15% higher than corresponding static values. Vibration tests showed a damping capacity for the aluminum span more than twice as great as for steel span; difference is attributed partly to greater number of splices used in aluminum span. F. Stüssi, Switzerland

2273. H. J. Greenberg and W. Prager, On limit design of beams and frames, Graduate Div. of applied Math., Brown University, Providence, R. I., Tech. Rep. A18–1, 18 pp., 6 plates (1949).

Paper treats limit design problems which concern statically indeterminate beams or frames under the action of given loads. The limit moments are prescribed throughout structure, and the safety factor against collapse is required. Extremum principles which give upper and lower bounds for safety factor are established, and their application to iterative calculation of safety factors is discussed and illustrated in case of plane frameworks.
Courtesy Mathematical Reviews F. B. Hildebrand, USA

2274. Jacques Heyman, The limit design of space frames, Graduate Div. of applied Math., Brown University, Providence, R. I., Tech. Rep. A18–2, 22 pp. (1949).

Author extends application of Greenberg-Prager maximum-minimum method [see preceding review] to calculation of safety factors in the limit design of space frames. Procedure is outlined specifically, and is applied to an explicit example.
Courtesy Mathematical Reviews F. B. Hildebrand, USA

2275. B. G. Neal and P. S. Symonds, The calculation of collapse and shake-down loads for elastic-plastic structures, Graduate Div. of applied Math., Brown University, Providence, R. I., Tech. Rep. A11–41, ii + 55 pp. (1949).

Numerical methods for determining either plastic collapse load or shake-down load of a rigid frame, continuous beam, or truss, subjected to concentrated loads, are illustrated in detail in the case of a specific frame. Procedure involves a systematic reduction of a set of simultaneous linear inequalities, and is sufficiently flexible to permit efficient use of preliminary information.
Courtesy Mathematical Reviews F. B. Hildebrand, USA

2276. Dernedde/Müllenhoff, The Cross-method for stepwise calculation of continuous beams and frames (Das Cross'sche verfahren), 2nd ed., Berlin, Wilhelm Ernst & Sohn, 1949, vii + 107 pp., 85 figs., 30 tables. Paper, $6^3/_4 \times 9^1/_2$ in. Price DM 12.

Book presents a clear description of Hardy Cross method of moment distribution, but gives practically no new developments nor any information not well-known. Several pages of tables list stiffness and carry-over factors for a number of special cases of beams with variable moment of inertia.

After a brief introduction to the method, various examples of continuous beams and bents are discussed in detail. Calculations are then carried out for beams of nonuniform stiffness. Bents under side or unsymmetrical loads are then taken up and effect of side displacements is considered. Finally, rigid-jointed frameworks, Vierendeel trusses, and multistory bents are investigated. A known proof of convergence of method is included at the end. No mention is made of application of Cross procedure when axial loads are present, nor of possible applications in determination of buckling loads of trusses and frames. Bruno A. Boley, USA

2277. S. Woinowsky-Krieger, On statics and kinetics of girder grillage (in German), Ingen.-Arch. 17, 391–402 (1949).

An approximate solution is presented of the problem of a grid-type girder bridge having a large number of continuous transverse girders. Solution is based on following assumptions: (1) Loads are applied to longitudinal girder; (2) vibration characteristics of main girders are known; (3) torsional stiffness of transverse girders is neglected; and (4) bending stiffness of continuous transverse girders is replaced by bending stiffness of a continuous plate spread uniformly over main girders. Author states that, compared with an exact solution, method outlined in paper furnishes good results and is more practical for structures with a large number of transverse girders. It is useful when making preliminary estimates prior to an exact solution.

Solutions are presented for deformation and shear forces in girder grillage as well as for vibration frequencies of system under a moving load. Karl Arnstein, USA

2278. W. A. Andrews, Plastic bending of wide flange beams, Weld. Res. Suppl. 14, 309–310 (July 1949).

Paper deals with utilization of strength of structural members beyond elastic range. Strain is assumed to be proportional to distance from neutral axis, and stress distribution is obtained from the stress-strain curve of material. Stress distribution so found is approximately a distorted parabola. Assuming such a distribution, an equation for equivalent section modulus of a wide-flange section is obtained. W. T. Thomson, USA

2279. L. A. Beaufoy, Influence lines for pin-jointed redundant trusses, Engineering 168, 49–50 (July 15, 1949).

By an application of Muller-Breslau theorem, it is shown that method of equivalent elastic systems may be used for determination of influence line for forces in any member of a pin-jointed redundant truss. C. T. Wang, USA

2280. B. G. Neal, Plastic collapse and shake-down theorems for structures of strain hardening material, Graduate Div. of applied Math., Brown University, Providence, R. I., Tech. Rep. A11–40, 36 pp. (1949).

Sufficiency conditions are established for plastic collapse and for shake-down of a truss whose members exhibit limited strain hardening. Corresponding conditions for frame structures are also discussed without proof. Results obtained generalize previous results relevant to ideally plastic materials.
Courtesy Mathematical Reviews F. B. Hildebrand, USA

2281. V. Kolář, Differential equations of deformations (in Czech), Technický Obzor SIA, 57, 191–195 (Dec. 1949).

Results of theory are presented and applied to differential equations in structural analysis, with three examples [open-web (Vierendeel) girder under eccentric loading, multiple-story frame,

and influence lines of bending moments at supports of a continuous beam with any number of spans]. J. J. Polivka, USA

2282. A. **Bebr, Analysis of continuous arches on elastic piers by method of moment distribution** (in Czech), Technický Obzor SIA, 57, 185–191 (Dec. 1949).

Hardy-Cross method is used for determining horizontal thrusts in continuous spans from which all other internal forces necessary for design can be derived [references are made to Hardy-Cross' bibliography, Guldan, Cocchi, Strassner, Kolousek and O. Novák]. Merit of method is emphasized. In this special case, it is recommended for analysis that the structural system be divided in two portions, with central support fixed. The unbalanced internal forces are then distributed over these individual portions.

J. J. Polivka, USA

2283. **M. De Kruyf, C. van der Poel, and R. Timman, Calculation of stresses in road and runway constructions,** Proc. sec. int. Conf. Soil Mech. Found. Engng. IV, 181–189 (1948).

Paper describes a method for calculating stresses occurring in a road or runway structure under static loading. Regarding a wheel load as equivalent to a circular area of uniform vertical pressure applied at surface of runway, use is made of a modified Boussinesq equation to compute vertical stress on upper surface of concrete layer, regarded as rigid. Two cases, for single and dual wheel loading, are calculated in detail. The stresses and deflections at all points in concrete due to each individual circular area of stress are computed mathematically, regarding subsoil as a Westergaard medium, and total deflections and stresses in concrete for single or dual wheel loading on runway are obtained by summation. For a fixed total load it is shown that dual wheel loading gives a lower stress in concrete than that due to a single wheel.

J. D. Coleman, England

2284. J. **Vasta, Structural tests on the passenger ship S.S. President Wilson—Interaction between superstructure and main hull girder,** Trans. Soc. nav. Archit. mar. Engrs., no. 4, 39 pp. 1949).

Paper outlines structural tests made to determine interaction between aluminum superstructure and main steel hull girder Strain gages were placed at critical points throughout ship. Bending moment on hull was varied by shifting ballast. Computed change in stresses, based on elemental beam theory, due to this change in moment is compared with strain gage readings.

Investigation was divided into four parts: (1) Hull girder strength; (2) deckhouse behavior; (3) stress concentrations; and (4) comparison between model studies and ship tests.

Tests showed that: (1) In area of superstructure, plane sections do not remain plane. Measured stresses at upper extreme fiber of ship's section were only 26% of values predicted from elemental beam theory. (2) Deckhouse structure contributed a moderate amount of stiffness to main hull structure and reduced maximum stresses about 25% from those calculated by beam theory. (3) Stress concentrations as high as 4.6 were measured in structure. (4) Stress at strength deck of a ship's structure with a moderately long deckhouse can be predicted from empirical expression $\sigma = (Mc_1/I_1) \times (I_1/I_2)$ where M is applied bending moment, c_1 distance from neutral axis of main hull section only to strength deck, I_1 moment of inertia at midship of main hull girder alone, and I_2 moment of inertia of main hull plus deckhouse at midship.

Karl Arnstein, USA

2285. E. S. **Jenkins and C. D. P. Pancu, Dynamic loads on airplane structures,** Soc. auto. Engrs. Quart. Trans. 3, 391–409 July 1949).

Authors discuss effect of elasticity of an airplane upon stresses which occur under action of various types of dynamic loads. First half of paper is devoted to a description of such problems as landing loads, gust effect, flutter and maneuvering loads, with particular attention given to methods of calculating response of structure to these loads. Necessary approximations and assumptions are discussed.

Second half of paper gives solutions for a seaplane landing impact and a gust analysis. Time history of reaction force in seaplane impact is represented by an analytic function of time, independent of motion, in order to linearize the problem. Accelerations in individual modes are computed and used to determine inertia forces. Two rigid-body modes and three natural elastic modes are employed. The natural modes involve wing twisting, both vertical and chordwise bending of wing, and vertical fuselage bending.

In gust analysis, effect of gust loads on tail as well as on wing are considered. Two rigid-body modes and two elastic modes are employed. Unsteady aerodynamic forces are simplified by assuming a constant chord and neglecting wing taper. [Further advancements in dyanmic load problems are contained in a contract report submitted to Bureau of Aeronautics, Navy Department, by R. L. Bisplinghoff, An investigation of stresses in aircraft structures under dynamic loading, M.I.T. (Jan. 1949).]

Stanley U. Benscoter, USA

2286. **Fred Werren, Fatigue of sandwich constructions for aircraft,** For. Prod. Lab. Rep. no. 1559–H, 5 pp. (1949).

A limited number of tests were made at Forest Products Laboratory to determine shear fatigue properties of an assembled sandwich panel with aluminum facings and perforated-aluminum-foil honeycomb core material. Results of tests and the S-N curves obtained from them are presented. All tests were made in accordance with methods described in Forest Products Laboratory Report No. 1559-A.

This is one of a series of reports intended to offer a comparison of shear fatigue properties of different sandwich materials.

Nicholas Di Pinto, USA

2287. **S. Jampel, An analysis of groups of piles, parts I & II,** Concr. constr. Engng. XLIV: no. 7, 201–208; no. 8, 253–257 (Aug. 1949).

Paper presents another exposition of method, developed by Swedish engineers, of analysis of loads in two-dimensional pile groups. Lateral pile resistances and end moments are first ignored and later are allowed for, by means of imaginary piles.

An element of novelty is introduced in presenting results of approximate derivation for coefficient of lateral soil reaction, which determines lateral pile resistance. An example of application of theory to a specific pile arrangement is given. From a practical viewpoint, paper may be criticized for too much emphasis on pile loads and ignoring significance of pile head displacements.

Alexander Hrennikoff, Canada

2288. **O. D'Heygers, Method of calculating tied arches** (in French), Publ. int. Assn. Bridge Struct. Engng. 9, 153–180 (Nov. 1949).

Analysis of bowstring girder bridges is considered by author on basis of assumptions similar to those made by Haviar [same source; see REV 3, **1918**], except that both arch and tie are here assumed to be of parabolic shape but with variable moment of inertia. Problem is solved in two stages. First, a basic system is analyzed consisting of a two-hinged tied arch whose deformation due to axial loads is ignored for all members including the suspenders; horizontal thrust and influence lines are calculated by the usual methods. In second stage, bending moments,

transverse and direct forces are corrected for variability of moment of inertia, axial deformation of all members, and variation of arch curvature.

These corrections can be found graphically or by numerical methods for which equations are derived, leading to rather lengthy calculations in practice. Method is illustrated by application to an arch with 12 panels and appears to be more cumbersome than that by Haviar. Its practical use is also considerably restricted by assumptions of parabolic shape for arch and tie, and of a constant modulus of elasticity for all members, which is only justified for steel bridges but not for reinforced concrete or mixed constructions where the modulus may vary considerably. G. G. Meyerhof, England

2289. G. De Kazinezy, The limit design of shells, Publ. int. Assn. Bridge Struct. Engng. Final Rep., 615–621 (1948).

Importance of taking into account rock deformation for design of arched dams is outlined by author. A method of design, due to Professor Ritter, considering this fact is briefly described.

From author's summary

2290. N. J. Hoff and Paul A. Libby, Recommendations for numerical solution of reinforced-panel and fuselage-ring problems, Nat. adv. Comm. Aero. Rep. 934, 29 pp. (1949).

Components of airplane structures, such as reinforced panels and fuselage rings, are considered. Deformed configuration of structures is assumed to be described by a finite number of degrees of freedom. These degrees of freedom usually represent linear or angular displacements of joints. Equilibrium equations associated with each degree of freedom are written in terms of displacements. A simultaneous solution of these equilibrium equations constitutes a solution of the problem. Paper discusses several methods of approximate solution of the equations: Relaxation method, matrix-calculus method, growing-unit method, Niles tables, and an electric-analog method.

The following conclusions are drawn, based on authors' extensive experience in these matters: (1) Relaxation process is advantageous in most reinforced-panel problems. (2) Electric analog is recommended when several similar panels with many loading conditions are to be analyzed. (3) Ring problems are best solved by matrix methods. (4) In very complicated ring problems a combination of matrix methods with growing-unit and relaxation methods may become advisable.

R. L. Bisplinghoff, USA

2291. F. J. Plantema and A. C. De Kock, The elastic overall instability of sandwich plates with simply supported edges, Nat. Luchtlab. Amsterdam Rep. S.346, 18 pp. (1949).

Energy method for calculating buckling loads of sandwich plates, developed by Plantema, is applied to a simply supported, infinitely long plate subjected to shear. Results are compared with those obtained in previous papers and are presented in simple graphs. Nondimensional interaction curves for combined compression or tension and shear are given and, finally, all results obtained by means of energy method are recapitulated.

G. V. R. Rao, USA

2292. John E. Duberg, A numerical procedure for the stress analysis of stiffened shells, J. aero. Sci. 16, 451–462 (Aug 1949).

In stiffened shells, such as used in aircraft structures, evaluation of stresses involves great mathematical difficulties for special deformations. So do problems of "shear lag," which are caused by tendency of a cross section to warp out of its plane in special cases of bending, and that of determination of bending stresses due to variation of torque in case of longitudinal torsion. These have not yet received a general practical solution. Author develops a numerical method for solving problems of this kind. Approximating the cross section by a polygon, and under usual assumptions, author establishes numerical equations for axial displacements of cross section. Iteration is applied for solving these equations and a set of special axes, "principal shear axes," are referred to in order to get rapid convergence.

Application to a simple case illustrates use of this method, and a more complicated problem is summarized in appendix.

Aurel A. Beleş, Rumania

2293. A. van der Neut, Design of structures on the basis of assumed deformations, Eng. Struct., 229–240 (Dec. 1949).

The usual method for design of redundant structures starts with an estimate of structure's dimensions and, by means of conditions of equilibrium and compatibility of strains, stresses developed within structure are determined.

The new method, proposed in paper, starts with an estimate of deformations and determines dimensions of structure by means of conditions of equilibrium.

Although the method is of great interest, especially for statically highly indeterminate structures, as it avoids complicated calculus, its use is limited by possibility of a correct estimation of deformations, by diversity of load distribution, by type of structures, etc. Author describes method in general terms and gives an application for design of a wind tunnel formed of a cylinder and a coaxial cone shell. Aurel A. Beleş, Rumania

2294. H. W. F. C. Héman, Calculation and manufacture of tubes in prestressed concrete (in Dutch), Ingenieur Indonesie 6[1] no. 29, 55–60; no. 31, 65–70 (1949).

Known principle of pipe design is described, whereby precast sections of concrete tubes with embedded longitudinal reinforcement are wrapped with spiral rod reinforcing under tension against cylindrical concrete elements. In practice, prestressing of spiral wire is not high and stable enough to prevent cracking in concrete due to shrinkage and high pressure in tube. These phenomena are investigated with aid of numerical examples calculated on basis of a relatively simple method and experimental observation. Under given conditions, including internal pressure, it is possible to determine the minimum initial tension of wrapping which secures adequate compression in tube. During manufacture caution must be taken that variations in prestress are small and that lowest intensity of prestressing remains a good deal higher than minimum tension resulting from calculation [about 20% is recommended, consisting of approx. 15% due to shrinkage and plastic flow of concrete, 3.5% and 0.5% due to elastic deformation of concrete and steel, respectively]. J. J. Polivka, USA

2295. J. Rotta, On the statics of a tire (in German), Ingen.-Arch. 17, no. 1 & 2, 129–141 (1949).

A study of deformations and stresses developed in a tire during rolling would be of great importance, but mathematical complications involved in such a study are quite inextricable. Therefore author considers only statical aspect of problem. Even in this case special assumptions are made concerning initial form and deformation of tire. Membrane theory of shells is used and conclusions are drawn about the forces acting on tire, especially about those developed on the contact surface with the ground. An interesting conclusion is that this last force depends only in a small measure on internal pressure of tire.

Author states that no comparison can be made with experimental data for car tires; the few observations made for aircraft wheels agree fairly well. Although there is a great difference between reality and author's assumptions, paper has great interest as a first step in the solution of this difficult problem.

Aurel A. Beleş, Rumania

Rheology (Plastic, Viscoplastic Flow)

See also **Revs. 2202, 2230, 2231, 2280, 2333, 2339, 2361**)

2296. **H. H. Hazlett, A. T. Robinson, and J. E. Dorn, An evaluation of a theory for plastic flow in anisotropic sheet metals,** Trans. Amer. Soc. Metals 42, 1326–1356 (1950).

Paper reports measurements of longitudinal and transverse strains in uniaxial tension tests on strips cut from anisotropic sheets of steel, aluminum, and magnesium. Data is correlated with a theory relating strain-ratios to orientation of specimen in plane of sheet. Shear strains are also measured by use of gridded specimens.

It is concluded that theory is a useful first approximation when state of anisotropy does not change appreciably during deformation. R. Hill, England

2297. **Roger W. Peters, Norris F. Dow, and S. B. Batdorf, Preliminary experiments for testing basic assumption of plasticity theories,** Soc. exp. Stress Anal. 7, 127–140 (1950).

The great majority of theories of plasticity can be classified into two types. Deformation theories are characterized by assumption that for continual loading, state of strain is uniquely determined by state of stress, and flow theories by assumption that (infinitesimal) increment of strain is uniquely determined by existing stress and increment of stress. In both types, loading is considered to constitute change in stress state which alters the plastic part of strain. A third theory of plasticity is based on concept of slip. According to this theory, initial shear modulus of a material in plastic compression depends on relative magnitude of compressive and shear components of stress increment.

Experiments upon thin-walled cylinders of 14S-T 4 aluminum alloy loaded into plastic range in compression and then twisted, revealed the following: (1) Initial torsional stiffness was found to be relatively insensitive to applied axial compression, even up to values of compression sufficient to give a secant compressive modulus of 0.44 times the elastic compressive modulus. This conclusion is in disagreement with deformation theories and in agreement with flow theories and with slip theory. (2) Rotations of principal axes of plastic strain were less than assumed by deformation theories, and rotations of principal axes of plastic strain rate were greater than assumed by flow theories. (3) Magnitudes of plastic strains were in good agreement with slip theory in a test performed at relatively low compressive stress, and in less satisfactory agreement with slip theory in a test performed at relatively high compressive stress ($E_{sec}/E = 0.80$ and 0.44, respectively, at beginning of twist). E. Siebel, Germany

2298. **C. A. Bouman, Lattice and space stresses as auxiliary quantities in the theory of fracture of ductile materials** (in Dutch, with English summary), Ingenieur 's-Gravenhage 61, W 53–59 (July 8, 1949).

Author surveys the theories of H. Brandenberger concerning "lattice and space stresses," discussed in a number of papers by Brandenberger (REV 1, **284, 456**) and more recently in the book *Neue Theorie der Elastizität und Festigkeit* (REV 2, **1263**).

A. van Wijngaarden, Holland

2299. **H. Hencky, Mathematical principles of rheology,** Reprint from Research 2, 437–443 (1949).

Author obtains "universal equations of rheology." His claim that paper "demands from reader some concentrated penetrating thinking" is substantiated by scarcity of logical definitions and reasoning, and by superfluity of confusing misprints. The notation changes in middle of paper; reviewer uses author's second system here. After an incorrect statement of history of finite strain theory, author introduces "projective strain and rotation" e_{ij} and ω_{ij}, which are respectively the symmetric and skew parts of $v_{i,j} + c_i v_j$, where v_j is velocity vector and c_i is an unknown vector. If parameter t used in definition of v_j is time, e_{ij} and ω_{ij} have the dimension of rates. Author's general dynamical equations are of incremental or rate type, although in treating case of simple extension he unexpectedly interprets e_{ij} as his earlier logarithmic finite strain measure [Z. Techn. Phys. 9, 214–223 (1928), etc]. Let σ_{ij} be an arbitrary stress distribution satisfying Cauchy's dynamical equation $\sigma_{ij,i} = \mu \delta v_j/\delta t$, where $\delta/\delta t$ denotes material derivative. Then author proposes

$$(\delta\sigma_{ik}/\delta t + \sigma_{i\lambda}\omega_{\lambda k} + \sigma_{k\lambda}\omega_{\lambda i})_i = \mu \delta^2 v_k/\delta t^2,$$

$$\delta\sigma_{ik}/\delta t = s_{ik} - \sigma_{ik}e_{\lambda\lambda} + \tfrac{1}{2}(\sigma_{i\lambda}e_{\lambda k} + \sigma_{k\lambda}e_{\lambda i}) \qquad (^*)$$

where for elastic bodies s_{ik} is given in terms of projective strain e_{lm} by usual linear Hooke-Cauchy law, while for fluids a Maxwellian relaxation term is added. The vague considerations motivating these proposals are not understood by reviewer.

Author applies his result to case of a cylinder stretched by internal pressure. Rate or increment equations similar to, but more plausible than Equation (*) were proposed by Zaremba [Bull. Int. Acad. Sci. Cracovie (1903), 380–403]; Jaumann [S.-B. Ak. Wiss. Wien 120, (IIa), 385–530 (1911), see § IX]; Handelman, Lin and Prager [Q. appl. Math. 4, 397–407 (1947)]; and Murnaghan [*Nonlinear Problems Mechan. Continua* (1947), N. Y. (1949), 158–174]. C. A. Truesdell, USA

2300. **D. C. Drucker, A reconsideration of deformation theories of plasticity,** Trans. Amer. Soc. mech. Engrs. 71, 587–592 (July 1949).

In case of a general stress it is assumed that permanent strain tensor E'' is still determined by instantaneous stress tensor Σ if, in addition, a criterion is used of what is meant by loading and unloading. In case of an isotropic material the hypothesis is that E'' increases somehow if some appropriate scalar function $f(\sigma_1, \sigma_2, \sigma_3)$ (σ_i = principal stress) increases, while no change in permanent strain occurs if f decreases or remains constant; dependence between Σ and E'' is such that principal directions of both tensors always coincide. For reasons of continuity a small change of f should produce a small change of E''.

In paper it is shown these postulates lead to contradiction. Assume, e.g., that the change of Σ is such that principal directions change by finite angles while increase in f is zero or very small—due to small changes in the σ_i. Then, principal directions of E'' will also rotate, and consequently E'' changes appreciably while df is arbitrarily small, and thus above postulate of continuity is violated.

Similar contradictions may obviously be reached in various ways. Author suggests experiments illustrating such inconsistencies. On the other hand, for certain very special changes of Σ (e.g., for "radial" change $\Sigma = K\Sigma_0$ (K a scalar constant, Σ_0 a constant tensor) which closely resemble the one-dimensional case, the "deformation theory" is consistent.

In flow theories where stress determines some rate of strain, such difficulties do not occur. Hilda Geiringer, USA

2301. **F. D. Stockton and D. C. Drucker, On fitting mathematical theories of plasticity to experimental results,** Graduate Div. of applied Math., Brown University, Providence, R. I., Tech. Rep. A11–44, 21 pp. (1949).

Authors show that existing theories of plasticity cannot be made to fit satisfactorily a certain set of experimental data. Several possible explanations of this failure are advanced.

Courtesy Mathematical Reviews F. B. Hildebrand, USA

2302. G. N. White, Jr. and D. C. Drucker, Effective stress and effective strain in relation to stress theories of plasticity, Graduate Div. of applied Math., Brown University, Providence, R. I., Tech. Rep. A11–42, 36 pp. (1949).

Authors propose certain definitions of effective stress and effective strain on basis of recent stress theories of plasticity. The effective stress is essentially uniquely defined in terms of a loading function associated with given material. On the other hand, effective strain may be defined either under assumption that dissipated work depends upon the state of stress alone, and not on the path, or by taking into account stress-strain relations. It is shown that these two alternatives lead to definitions which are in substantial agreement in certain cases, and that comparisons in other cases may be obtained experimentally.

Courtesy Mathematical Reviews F. B. Hildebrand, USA

2303. A. Graham, A new approach to the problem of flow, Proc. seventh int. Congr. appl. Mech. 1, 348–351 (1948).

To separate elastic deformation and plastic flow occurring at the same time, author uses a frame of reference that deforms with body in its elastic deformation. However, he does not give a rigorous definition of this frame nor indicate form of equations of motion or equilibrium in deformable frame.

Author seems to consider the irrotational part of deformation to be wholly elastic, so that deformation of frame should follow that part of deformation of body; reviewer does not think this assumption is valid.

It is shown, in conclusion, that for small deformations there is a formal identity of equations with those of electromagnetic field. It is expected that this would allow study of such cases as fatigue or damping of small motions through plastic deformation.

Jean Goguel, France

2304. Jay W. Fredrickson and Henry Eyring, Statistical rate theory of metals—I. Mechanism of flow and application to tensile properties, Metals Technol. 15, T.P. no. 2423, 33 pp. (Nov. 1948).

Rate process theory developed earlier by Eyring for viscous flow and plastic behavior is extended to account for true stress-strain curve for metals in tension. A model is proposed which consists of a finite number of Maxwell units of elastoviscous type. These units differ from one another only in values of constants involved. The stress-strain curve is broken up into three regions: (1) From point of initiation of stress to yield point; (2) from yield point to maximum load; and (3) from maximum load to point of fracture. Stress-strain relationship is worked out in terms of these units and the constants evaluated. Effect of temperature and strain rate is found, and authors show that by suitable values of constants, calculated curve can be made to agree with experimental one. Values of parameters required to fit experimental data indicate that plastic deformation occurs by movement of rather large domains from one position to another in their model. Free energy of activation and volume of domains comes out to be linearly dependent on temperature. Also, deformation appears to be almost an entropy effect.

Edward Saibel, USA

2305. R. Furth, On the theory of strength of quasi-isotropic solids, Phil. Mag. 40, 1227–1233 (Dec. 1949).

A thermodynamical theory of strength of solids has been previously proposed by author without reference to a detailed mechanism. Principal idea is assumption of a close relationship between phenomena of breaking and melting, melting being identical with breaking of a solid structure by thermal movement of atoms, and vice versa, breaking being nothing else than melting enforced by an applied stress. Author points out in a simple way that his theory is related to Bragg's theory of strength of metals, which is based on assumption of existence of a lattice block structure. Explanation of such a corelationship is to be sought, according to author, in hypothesis that melting consists in complete breakup of a lattice into separate blocks. As heat of melting is a fixed quantity for a given substance, author concludes that this block structure must be an intrinsic property of solid crystalline material, in contrast to "mosaic structure" which strongly depends on previous mechanical and thermal treatments of specimen. Something like an intrinsic block structure of perfect crystals was previously suggested by Born because of certain difficulties met in application of quantum mechanics to dynamics of crystal lattices.

Author points out, further, that size of crystal blocks, as determined with X-ray measurements by Wood and collaborators on progressively deformed crystals of annealed metals, has same order of magnitude as critical block size calculated from Born's theory of a perfect crystal lattice. T. S. Kê, China

2306. A. J. Kennedy, The effect of instantaneous pre-strain on the character of creep in lead polycrystals, Proc. phys. Soc. Lond. Sect. B 62, 501–508 (1949).

Paper reports some careful tests in which time-extension curves for lead wires are obtained at constant stress following varying degrees of rapid prestrain. As degree of prestrain is increased, extension for given times decreases, while slope of time-extension curve becomes more nearly constant. Author interprets these observations in terms of concepts of Andrade, whereby creep comprises two distinct components, a β or transient (slip) flow, and a κ or permanent (viscous) flow. The Andrade creep equation then can be expressed as $l = l_0 (1 + \beta t^{1/3}) \exp(\kappa t)$ when l and l_0 are instantaneous and initial lengths of specimen and t is time. Effect of increasing prestrain, then, is such as to absorb more and more of transient portion of flow process so that ensuing creep becomes more and more solely a viscous phenomenon. By shifting time scale and replacing t by $t + t_0$ where t_0 is a constant, but keeping β and κ same, Andrade equation fits experimental data for prestrains of less than 10%. Temperature of test was in all cases 19.8 C.

Additional experiments are performed in which prestrain temperature is also changed (100 C, 35 C and −180 C). They show that prestrains of about 5% are fairly insensitive to variations in prestrain temperature, while a marked change, similar to increasing degree of prestrain occurs at 10% as prestrain temperature is lowered. Louis F. Coffin, Jr., USA

2307. Clarence Zener, Relaxation phenomena in metals, Physica 15, 111–118 (Apr. 1949).

Author gives a survey of what has been learned about laws governing relative motion of atoms in metals by study of mechanical relaxation. Relaxation spectrum usually shows several peaks, each of which refers to a distinct relaxation process. Especially mentioned are relaxation phenomena due to migration to preferred sites of interstitially dissolved atoms in body-centered cubic metals [carbon and nitrogen in ferrite (Snoek)], orientation in a preferred direction of axis of nearest-neighbor-pair of substitutionally dissolved atoms, and self-diffusion in grain boundaries of pure polycrystalline metals. It is pointed out that motion of dislocations in crystals will cause a relaxation effect which may give information about their properties. Same applies to motion of holes in lattice and the part they play in diffusion process. Method allows also for the study of initial stages of precipitation from supersaturated interstitial solutions [nitrogen in ferrite (Dijkstra)]. J. H. Palm, Holland

2308. Piero Giorgio **Bordoni, Theory of elastic dissipation in monocrystals according to quantum mechanics: a new relaxation effect** (in Italian), Ric. Sci. 19, 851–861 (Aug. 1949).

Author states that causes which are commonly attributed to dissipation of energy, namely, thermoelastic and magnetoelastic effects, viscous behavior of region between crystalline grains, etc., are not sufficient to account for phenomenon of energy dissipation in longitudinal vibrations of nonmagnetic single crystals. In paper there is given a treatment of problem by use of statistical mechanics and the assumption of pre-existing dislocations. Time of relaxation and its dependence on temperature are derived. In case where crystal is acted on by an oscillating force, following results are obtained: (1) Logarithmic decrement of oscillations reaches its max. at a temperature T_m generally lower than Debye temperature when oscillations are of acoustical frequency. (2) Modulus of elasticity tends to a limiting value, independent of presence of dislocations and independent of past history, when temperature tends to absolute zero. (3) For T greater than T_m, modulus of elasticity increases with a treatment which eliminates dislocations, and conversely.

Experimental work done by author on Pb, Al, Cu, and Ag is cited to confirm theoretical findings. Edward Saibel, USA

2309. **W. Soete, Measurement and relaxation of residual stresses,** Weld. Res. Suppl. 14, 354–364 (Aug. 1949).

Author classifies residual stresses as (*a*) reaction stresses (developed by welding under restraint); (*b*) macroscopic stresses developed by nonuniform plastic deformation; and (*c*) microscopic stresses which vary from grain to grain. Paper deals with measurement and possibilities of relieving residual stresses of type (*b*) in welded plate. By placing three electric-resistance strain gages radially about a point at 120-deg angles with each other, measured strains, when a small hole is drilled, are interpreted in terms of residual stress at the point (Mathar's method). Emphasis is placed on residual stresses arising from localized plastic deformation. Relief of residual stress requires elimination of elastic deformations, which may be achieved in two ways: (1) Stress-relieving heat treatments (heating entire member to temperatures sufficient for yield point of metal to decrease to values near zero). (2) Application of a state of stress opposite in sense to existing state of stress (by localized heating or by mechanical operations).

Data are presented from tensile tests on longitudinal and inclined weld seams showing relative strain in weld bead as compared with the base metal, and the resulting change in residual stress due to the loading cycle. Tensile deformation in the longitudinal direction of weld bead did not increase in proportion to strain in parent metal; at a stress of about 16.5 tons/sq. in., deformation changed sign and began to decrease. This was attributed to automatic unloading of weld as parent metal developed localized flow, allowing residual stress and strain in weld to decrease. Mechanical processes such as cold-rolling or peening are discussed as methods of superposition of a compressive residual stress. Comprehensive investigation of methods of relief of residual stresses is suggested with the thought that other methods may prove more efficient and less drastic to material than heating whole member. T. J. Dolan, USA

2310. F. R. **Eirich, Report on fundamental experimental methods in rheology,** Proc. int. rheolog. congr. Holland, IV, 3–42 (1948).

In author's words, report deals with "some more recent methods which have come to (his) notice with which he is more familiar and which, in his opinion, possess especially recommendable features and ought to be more widely known or applied." With these self-imposed restrictions, report is very comprehensive, and introduction gives a compact picture of the continuum, molecular, and mechanical-model approach to rheology. Regarding latter, reviewer would point out that it is not limited to simple proportionality of stress. It should be kept in mind that model named there "Voigt body" has by others been named "Kelvin body." In second part, useful remarks about experimental procedure and precautions are made. Methods dealt with are: (1) extension and compression; (2) laminar shear of rotational, torsional, and telescopic type; and (3) application of sinusoidal stresses. Discussion of latter is especially valuable. Reviewer would point out that, in extension, sample need not be strong enough "to support its own weight." Many investigators, following Trouton [Proc. phys. Soc. 19, 47–56 (1905)], have used, as samples, quite weak prisms floating horizontally on mercury, and sometimes, in addition, immersed in a liquid of even density and so made weightless.

Third part deals with a number of lesser known methods, among them Tobolsky's tilting relaxometer and another one which measures stress and birefringence simultaneously; Mooney's automatic extensometer in which rates of loading are controlled against a calibrated spring; Guth's automatic compression relaxometer; and, in greater detail, with a three-cylinder differential viscometer developed at Brooklyn Polytechnic Institute. Especially valuable is the discussion of sinusoidal-stress applying apparatus including (*a*) application of external vibrating masses, (*b*) wave propagation, and (*c*) resonance methods. Report is slightly marred by many misprints in reference numbering. Reader will be able to correct some, but one (39) is decidedly misleading. Actually, method of plotting "consistency-variables" and interpretation of deviations as pointing to wall effects is due independently to Reiner [J. Rheology **2**, 337–350 (1931)] and Schofield and Scott Blair [J. phys. Chem. **35**, 1212–1215 (1931)]. With regard to disk-and-cone instruments, statement, made by different authors, that velocity gradient is constant is repeated, but it can be shown that this is only approximately correct for very small angles, and that if angle between cone and disk is, say, 15 deg. velocity gradients differ by as much as 6%.

M. Reiner, Israel

2311. **W. Lethersich, A theory of the transition from tough to brittle fracture with special reference to velocity effects in impact testing,** Proc. seventh int. congr. appl. Mech. 1, 61–73 (1948).

The energy to fracture, and tensile strength are discussed in terms of a very simple model, with fracture obeying maximum strain laws for both brittle and tough behavior. For brittle fracture, it is assumed that the hydrostatic component of the state of stress controls, while the shear component controls ductile behavior. Vastly idealized. Maxwell Gensamer, USA

2312. **W. Schaad, Theory of avalanche protection** (in German), Schweiz. Bauztg. 67, 431–433 (1949).

See comment on dissertation on this subject by E. Bucher (REV **2, 1470**). An alternative formulation is offered for analysis of flow in standard case of snowbank initially uniform in depth, extending over edge of a horizontal field and down a uniform slope to a retaining structure. Relations in this standard case are exhibited in a diagram and pertinent generalized formulas, both according to Bucher and as now modified, are assembled in a table. W. P. Roop, USA

2313. **Ch. Boulanger, The Portevin-Le Chatelier phenomenon in mild steels,** Génie civ. 126, 406–408 (1949).

Author describes tensile tests (velocity = 0.32 in./min on

wires, diam = 0.001″, length = 1 in.) of soft steel and iron with very high degree of purity; influence of testing-temperature on the course of stress-strain diagram is also investigated. It is shown how the irregularities in the stress-strain curves are influenced by cold-working and aging or by impurities (carbon, nitrogen, oxygen) in the temperature range of blue brittleness.

M. Hempel, Germany

2314. O. Föppl, Residual stresses in surface-rolled rods of circular section (in German), Z. angew. Math. Mech. 29, 20–21 (1949).

Elastic and plastic deformations induced in a circular rod by surface rolling are determined by repeatedly boring out the rod and measuring its dimensions. In order to simplify analysis, Poisson's ratio is assumed to be zero, and rod to be of infinite length and composed of an elastic core surrounded by a uniformly plastically deformed layer. Paper is a condensed report of technique used, which is described fully in *Mitteilungen des Wöhler-Instituts, Heft 42.*

Ch. Massonnet, Belgium

2315. M. Roš and A. Eichinger, Stress and breaking point of solid bodies (in German), Schweiz. Bauztg. 67: 383–387; 395–401 (July 1949).

First part deals mainly with theory of flow and fracture strength of metals as presented by authors in 1929 (E.M.P.A. Diskussionsbericht no. 34). It is assumed that a single-valued relation exists between octahedral shearing stress and corresponding displacement of center of gravity of octahedral plane, independent of state of stress. Authors arrive at the known stress-strain relations $\delta_1 = D^{-1}[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)]$ with similar expressions for δ_2 and δ_3. Variable plastic-modulus D is equal to ratio of stress and strain in uniaxial tension at corresponding octahedral stress. According to authors, the tests, carried out at E.M.P.A., are in good agreement with this theory.

In second part, recent theory of Brandenberger (REV 2, **1263**), in which stresses are divided into volume and lattice stresses, is discussed. Their main objections can be summarized as follows: In the primary elastoplastic region, stresses and strains are calculated according to theory of elasticity, which is not justified. Consequently, stress-strain curves should be discontinuous. Theory is not in agreement with experimental data on elastic limit, yield limit, and fatigue strength. In reply, Brandenberger remarks that in the primary elastoplastic range, laws of elasticity are applied in that principal direction where yielding has still not occurred. Brandenberger further points out that stresses and strains in this range can be easily calculated even in a simple graphical way, and are in good agreement with experimental data.

In their final reply Roš and Eichinger repeat that Brandenberger's theory is not in agreement with the real behavior of metals, and emphasize that Bauschinger effect cannot be calculated on the basis of elastic constants, as would follow from this theory.

J. H. Palm, Holland

2316. V. V. Sokolovskiĭ, Plane and axisymmetric equilibrium of a plastic mass between rigid walls (in Russian), Prikl. Mat. Mekh. 14, 75–92 (1950).

Stress-strain relations of plastic deformation theory are used with the hardening law given by $S = KE^{\mu}$, where S and E are stress and strain intensities, respectively, and K and μ are constants. Assuming that only radial displacement takes place within the wedge, the components of strain are expressed in terms of E and a variable ψ which is a function of wedge angle θ only. Differential equations for ψ for both plane and axially symmetric wedge are derived. Boundary values for ψ are obtained for $\mu \neq 0$ under the assumption of adhesion at the walls, and for $\mu = 0$ under the assumption of a constant friction force at the walls. Numerical solutions are given for $\mu = 1/2$ and $1/3$ for the plane wedge, and $\mu = 0$, $1/3$ and $2/3$ for the axially symmetric wedge. A solution in closed form is given for $\mu = 0$ for plane wedge.

Courtesy Mathematical Reviews H. I. Ansoff, USA

2317. R. V. Southwell, On the computation of strain and displacement in a prism plastically strained by torsion, Quart. J. Mech. appl. Math. II, 385–397 (Dec. 1949).

Paper is concerned with plastic strain in a prism subject to pure torsion. The material is assumed to be nonwork-hardening, and to satisfy Prandtl's stress-strain law which relates stress to increments of plastic strain. Author verifies known fact that for torsion problem this law reduces to one which relates stress to total plastic strain.

Extent of plastic straining is measured by the quantity

$$(A - 1)\gamma_{YY},$$

where $A = 1 + 3\lambda\mu$. Here 3λ is the ratio of plastic strain to stress, μ is modulus of rigidity, and γ_{YY} is the shear strain at a point, the instant that point first became plastic. Partial differential equations and boundary conditions for A are derived, and equations are given for determining the warping once A is known. These equations are valid for case where prism is only partly plastic, and are not restricted to simply connected domains.

Equations are solved by relaxation methods for the hollow rectangular prism with quarter-circle fillets whose stress solution was obtained by Shaw [Australian Council for Aeronauties, Rep. ACA-11]. No details of method of computation are given, but figures are presented showing the level curves of $A - 1$ and of warping at two different stages of twisting process. Author points out fact that although stress in the neighborhood of the fillets reaches its limiting value for a comparatively low torque, thus denoting intense stress concentration, shear strain increases slowly and does not double in value until prism is almost fully plastic.

Philip C. Hodge, Jr., USA

2318. M. Reiner, Relation between stress and strain in complicated systems, Proc. int. rheolog. congr. Holland, IV, 44–65 (1948).

Objective of paper is to show where rheology stands at present in the attempt to come ever closer to stress-strain behavior of actual materials. "Complicated systems" of the title are systems for which neither simple Newtonian liquid, simple Saint Venant plastic solid, nor simple Hookean elastic solid provides a sufficient approximation.

First complication treated is variability of rheological coefficients—change of modulus of elasticity with strain, of coefficient of viscosity with strain rate, and of plastic strength with strain (strain hardening). Second complication is the combination of fundamental properties, e.g., relaxation of elastic stresses. Stress-strain behavior of various bodies is represented by series, parallel, and more complex combinations of springs, dashpots, and solid friction devices (e.g., a weight on a platen). Included in discussion are such subjects as volume flow, volume plasticity, viscous Poisson ratios, and dilation due to shear.

S. B. Batdorf, USA

2319. H. Fromm, On the theory of structure formation at solidification of pure metals (in German), Z. angew. Math. Mech. 29, 10–11 (Jan.-Feb. 1949).

Statistical and geometrical aspects of solidification of metals is a known problem which has been tackled often with varying

degrees of success. Author considers problem anew and makes several assumptions. One is that near melting point, rate of growth of a solid is proportional to amount of undercooling. Further, at low temperatures, rate of nucleation decreases in an exponential manner. Author then takes up the whole process of crystallization from beginning to its completion and obtains a corresponding integral equation. Finally, he takes into account the heat balance by considering heat of nucleation, heat of crystallization, and necessary specific heats. Under assumption of a constant temperature, the final equation can be solved.

Reviewer believes that the work, although mathematically correct, makes rather drastic physical assumptions which make practical applicability rather questionable. It is known that crystallization of pure metals is geometrically a complicated process and that crystalline anisotropy plays an important role in determining rate of growth and ultimate distribution of grain sizes. It would be interesting to make a detailed comparison with observations and to check validity of basic assumptions.

Roman Smoluchowski, USA

Failure, Mechanics of Solid State

(See also Revs. 2298, 2307, 2312, 2315, 2340, 2361)

2320. **V. Ya. Shekhter, Theory of the diagram of additive deformations and calculation of the true resistance to failure** (in Russian), Zavod. Lab. 15, 957–961 (Aug. 1949).

Author defines the additive elongation φ by equation

$$\varphi = \int_{l_0}^{l_1} dl/l = \ln(l/l_0) = \ln(1 + \delta) = \ln[1/(1 - \psi)]$$

where l_0 is initial length, l_1 final length of specimen, δ and ψ are elongation and reduction of area in ordinary meaning. Tensile diagram is discussed in terms of φ.

Condition for maximum strength σ_B is $dS/d\varphi = S_B$ (S true stress). For beginning of necking it is found $S_B = \sigma_B e^{\varphi_B}$. S_B can be calculated if uniform elongation has been measured.

Experience shows that true stress at breaking point S_k and corresponding value of φ are approximately connected by relation $S_k = \sigma_B (1 + (e^{\varphi} - 1)\, \varphi_k/\varphi_B)$.

Calculated values of S_k differ from experimental data, even for different temperatures, by not more than about 5%. Only steel shows a difference of 20%. George Masing, Germany

2321. **André Gneussier and René Castro, Mechanical behavior of polycrystalline isotrope metals. Analogy of embrittling factors** (in French), Rev. Métall. 46, no. 8, 517–536 (Aug. 1949).

Paper considers phenomenological aspects of fracture problem in isotropic metals. Factors associated with tendency toward brittleness, namely, triaxiality, velocity of flow, and temperature, are discussed. Some experiments on notched bars of various steels are reported, parameters being bar-width, temperature, material, and heat treatment. Specific impact energy (total energy divided by notch cross section) is obtained by a pendulum-type impact testing machine. These tests reveal characteristic behavior common for mild steel, namely, decreasing specific energy for increasing width of bar and decreasing temperature.

Balance of paper reviews work of Ludwik, Kunze, and MacAdam regarding development of a flow curve and a fracture curve to explain fracture behavior in metals. Authors suggest that a fracture curve should in reality pass through maximum-load point in a true stress-strain test in simple tension. They state that this point is actually point of failure, and that changing constraint associated with necking phenomenon increases ability of material to deform further. This point of view may not find wide support from investigators in the field. Louis F. Coffin, Jr., USA

Design Factors, Meaning of Material Tests

(See also Revs. 2289, 2293, 2350, 2452)

2322. **S. Travers, Reflection of a shock wave in a resilience test of a traction specimen** (in French), Mémor. Artill. 23, no. 4, 847–858 (1949).

This is first of two memoirs by author devoted to an interpretation of rupture tests reported by Bertrand [same source, 22, no. 2 (1948)]. By considering a simplified model of frame in which specimen is mounted, author shows that elastic properties of mounting account for pressure discontinuities of observed order of magnitude during elastic stage of the phenomenon. Plastic stage will be discussed in a second paper.

I. N. Sneddon, England

2323. **G. Murphy, Evaluation of stresses and strains in axially loaded compressive specimens,** Proc. seventh int. Congr. appl. Mech. 1, 120–130 (1948).

Paper aims at presenting an analytical procedure for evaluation of compressive and shearing stresses in a conventional compressive specimen, taking into account frictional restraint developed at ends of specimen. Analysis represents an extension of the Cross method of moment distribution as modified to apply to problem of a homogeneous solid subjected to boundary stresses.

During whole process of carry-over and balancing operations, moment equilibrium of elements, into which specimen is divided, is disregarded, only horizontal forces being considered. Vertical forces on the elements are found afterward by equilibrium considerations. It seems, therefore, that there will be no compatibility of stresses and strains for final solution.

F. J. Plantema, Holland

2324. **F. Campus, Fundamental questions relating to welded constructions** (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale 2, 147–247 (1947).

Report of three lectures consists principally of a comprehensive survey of author's book on same subject [Recherches, études et considérations sur les constructions soudées, Liége (1946)]. It deals with results of a vast amount of experimentation which has been carried out to find an explanation for accidents with some Belgian bridges, and treats successively: (*a*) Strength, endurance, and type of fracture of welds, (*b*) shrinkage and distortion of welds, and (*c*) weld stresses, weldability of steels, and safety of welded constructions.

In author's opinion, small load variations in combination with severe weld defects and high static stresses due either to rolling, restraint, or defective mounting have caused the accidents. Brittleness as a possible cause for poor behavior of constructions is excluded. Material demonstrates its brittleness only after a crack has started, by preventing final fracture from being attended with plastic deformations. In order to study the occurrence of brittle fractures, a notched tensile test piece as well as a tensile test piece with a longitudinal weld bead has been used. Although these tests are not yet meant for specification purposes, they have been accepted as tentative standards.

To measure deformations of welds, special apparatus has been designed which makes use of a system of ball indentations as reference points. V- and X- butt welds, as well as fillet welds, have been investigated for various welding conditions and plate thicknesses, test pieces either being free or kept in a state of restraint. Corresponding stresses have been measured by trepanning methods. Author considers a steel weldable if it is apt to resist safely the shrinkage effects under actual conditions, weldability thus having no absolute meaning.

J. A. Haringx, Netherlands

Material Test Techniques

(See also Revs. 2310, 2323, 2324, 2336, 2347, 2350, 2353, 2354, 2355, 2357)

2325. Paul G. Nelson and Joseph Winlock, A method of determining the percentage elongation at maximum load in the tension test, Amer. Soc. test. Mat. Bull. no. 156, 53–55; 162, p. 46 (Jan. and Dec. 1949).

Using this method, determination of elongation at maximum load depends: (*a*) Upon fact that when "true" load is plotted against "true" strain, maximum load occurs at point where slope of true-stress true-strain curve is numerically equal to true stress at that point. (*b*) Upon assumption that, if logarithm true stress is plotted against logarithm true strain, a straight line is obtained. Stress and strain at a point prior to maximum load and remote from yield are observed together with maximum load. Strain at maximum load may then be calculated. Calculations, however, are very complex and method could not be recommended except where specimens are too short to apply double gage length method. J. A. Pope, England

2326. Carl Bihlmaier, Relationship between impact strength determined by using different sizes of test specimen (in German), Arch. Eisenhüttenw. 20, 31–35 (1949).

An experimental paper giving results of impact strength determinations using notched steel specimens of different dimensions, varying from Charpy pattern ($30 \times 30 \times 160$ mm^3 over-all) to Mesnager pattern ($10 \times 10 \times 55$ mm^3 over-all). A series of curves shows results for relationship between impact strength obtained with five different types of specimens, and results are compared with those of previous investigators.

R. M. Davies, Wales

2327. Nikolaus Ludwig, Contribution to the development of small notched impact test specimens (in German), Arch. Eisenhüttenw. 20, 27–30 (1949).

A description of a series of impact experiments with notched specimens of steel 55 mm long, 6 mm square section, with notches 2 mm deep and 0.75 mm diam, resting horizontally on supports 40 mm apart. Results are given for various types of steel at a number of temperatures ranging between −100 and +200 C. Results are compared with those obtained using DVM test specimens, 55 mm long, 10 mm square section, and notched to a depth of 3 mm. R. M. Davies, Wales

2328. W. Reinecken, Influence of the notch form and machining on the impact strength of steel at low temperatures (in German), Arch. Eisenhüttenw. 20, 37–40 (1949).

A series of experiments on impact strength of constructional alloy steels was carried out, the following factors being varied: (*a*) Composition of alloy, (*b*) temperature (between −40 and 20 C), (*c*) shape of notch, and (*d*) surface finish. Results show that variations due to changes in (*a*) and (*b*) are completely outweighed by variations in (*c*) and (*d*). R. M. Davies, Wales

2329. Ch. Massonnet, A new apparatus for determining the forces in plane, elastic elements (in French), Publ. int. Assn. Bridge Struct. Engng. 9, 321–340 (Nov. 1949).

Author describes his apparatus for determining forces within plane elastic disks of various shapes and subjected to various conditions of straining and deformation. New device works on principle of an integrator which gives results in form of equivalent forces on boundaries of considered disk.

Author has used device to obtain results for three connecting elastic members in a plane and to a section of an arch dam. Apparently, shear forces from adjacent sections of dam are neglected. Author proposes that his device will also give solutions to certain other problems involving interacting elastic elements. Apparatus should be of interest to those studying methods of experimental stress analysis. R. G. Sturm, USA

2330. C. H. Reichardt, H. Schaevitz, and J. H. Dillon, Stress-strain-time apparatus for fiber testing, Rev. sci. Inst. 20, 509–516 (July 1949).

Authors discuss design of a stress-strain apparatus for testing of fibers. Stress is measured by means of ring dynamometer in conjunction with a linear variable transformer. Apparatus may be designed to operate at either constant rate of strain or constant rate of stress. Load range for full scale coverage on recorder may be varied from 1 to 50 gms. Strain rate is varied by means of gear system. Frank J. Mehringer, USA

2331. J. H. Rondeel, W. K. G. Floor, and A. C. de Kock, Development and testing of an apparatus for calibrating extensometers and some preliminary calibrations (in Dutch), Nat. Luchtlab. Amsterdam Rep. S. 359, 26 pp. (Aug. 1949).

A shaft is suspended at two points by circular membranes held at ends of a surrounding metal cylinder, so that it can move only along its own axis; a system of gear wheels ending in a differential micrometer screw can move shaft over a length of 0.3 mm. One end of shaft carries a metal block to which is pressed one feeler of extensometer to be calibrated; other feeler is pressed against a block fixed to frame of calibrating device. Other end of shaft carries a glass disk perpendicular to axis of shaft; a parallel glass disk is fixed in frame of apparatus. Axial movement of shaft, produced by micrometer screw, is measured by observing interference fringes arising by reflexion of sodium light between glass disks. Two Huggenberger extensometers have been calibrated with the device; results are given and sources of error discussed.

E. Orowan, England

2332. Joseph Marin, A biaxial stress machine for the determination of plastic stress-strain relations, Proc. Soc. exp. Stress Anal. 7, no. 1, 71–82 (1949).

A biaxial-stress static-test machine is described, as are also results of tests on Alcoa 24S-T extruded-tubing specimens. To measure strains till rupture, electric resistance-wire strain gages are used in a novel arrangement as "clip" gages. Results are given in tables and curves, and are compared with theoretical calculations. A. N. Zamboky, USA

2333. G. P. Zaitsev, Brinell hardness as a function of plasticity parameters of metals (in Russian), Zavod. Lab. 15, no. 6, 704–717 (June 1949).

In the hardness test, diameter of indentation d is related to diameter D of the ball and applied load P by known experimental formula $P = a_0 d^n/D^{n-2}$ where a_0 (with dimensions of stress) and n (dimensionless) are characteristic quantities for material. Author attempts to relate these quantities to another set of quantities, tensile strength ($T.S.$) and uniform deformation, ϵ_0, in simple tensile test.

By means of an analysis of local and average deformation in indented area, he shows that a close parallelism exists between the diagrams: (1) stress-strain diagram in simple tension, and (2) $H/a_0 - d/D$ diagram in the hardness test, where H is Brinell hardness. On basis of experimental data on a variety of metals, following approximate relations are derived: $T.S. = 0.37a_0$, $\epsilon_0 = (n-2)/1.13$, and $(Y.P.) = (T.S.)\,(2 - n/2)(4\epsilon)^{(n-2)/2}$.

In last formula ($Y.P.$) represents the offset yield point for a given offset, ϵ (e.g., 0.2%). In order to obtain n and a_0 entering

above formulas, two hardness tests must be performed. For example, using two different balls 10 mm and 5 mm, two indentations are made under same load, P, d_5 and d_{10}. Then $n = 0.60206/(0.30103 - lg\ d_{10}/d_5)$ and $a_0 = (P/D^2)\ (D/d)^n$.

Daniel Rosenthal, USA

2334. **G. H. Vosskühler, Non-destructive testing of aluminum-magnesium-zinc alloy Hy 43 by magnetic-induction methods** (in German), Metall: no. 15/16, 247–251; no. 17/18, 292–295 (Aug. and Sept. 1949).

"Durokawimeter" consists of an exploratory coil carrying a-c current, to be placed on surface of a piece whose electrical resistance is to be determined. Input to coil, read directly on a dial, is a function of induced circulating currents in piece, and, so, of its resistance. Paper gives results of a large number of tests on aluminum-alloy samples, correlating dial readings with electrical resistance, hardness, and effect of various heat treatments. Instrument is suggested as a nondestructive means of inspection, within limited fields, but it is not described in detail.

M. F. Sayre, USA

Mechanical Properties of Specific Materials

(See also Revs. 2306, 2324, 2327, 2328, 2332, 2333, 2357)

2335. **E. J. Ripling, Tensile properties of a heat treated low alloy steel at subzero temperatures,** Trans. Amer. Soc. Metals 42, 439–454 (1950).

Tensile tests on nearly cylindrical test pieces of SAE 1340 at different low temperatures and in different conditions of tempering reveal a ductility minimum for tempering temperatures of 500–700 F if testing temperature is −220 F, or especially if it is −321 F. This ductility minimum is in same tempering range as room-temperature toughness minimum observed by other investigators. For high tempering temperatures, ductility is high and falls slowly with testing temperature; for low tempering temperatures, it is low and falls slowly; for intermediate tempering temperatures, ductility falls off abruptly below a certain subzero temperature, giving appearance of a "transition temperature."

J. D. Lubahn, USA

2336. **B. J. Lazan, A study with new equipment of the effects of fatigue stress on the damping capacity and elasticity of mild steel,** Trans. Amer. Soc. Metals 42, 499–558 (1950).

Damping capacity and dynamic modulus of elasticity of materials are extremely complex, continuously changing properties which are difficult to measure, as they are affected by a multitude of variables. Existing high stress damping and elasticity data are unreliable and often contradictory because all these variables were not isolated and controlled. Starting with the simple and known principle that lateral deflection of a rotating cantilever beam is proportional to its damping capacity, while its vertical deflection is a measure of its dynamic modulus, author developed new, reliable, accurate and flexible machine and new techniques that enable experimental measurements of these properties so as to show trends that could not be observed with previously available equipment.

Author used his equipment to test mild steel subjected to sustained cyclic stresses and reports that cyclic stress below 80% of fatigue limit has little effect on damping and elasticity, whereas stressing between this value and fatigue limit increases damping as much as 25 times and reduces modulus by as much as 11%. Cyclic stress above fatigue limit has even more pronounced effects. Author uses a new term, "cyclic stress sensitivity," to define these changes in materials. An analysis and correlation of changes in damping with changes in dynamic modulus is given. Consistent and significant pattern trends are presented to show a possible relationship of damping and elasticity to fatigue strength.

Author also reports on some exploratory tests on effects of stress history at various stress levels, overstressing, understressing, frequency of stress cycles and rest periods. For example (after prolonged constant cyclic stress), rest at zero stress causes 45% decrease in damping and 3% increase in dynamic modulus of mild steel under certain conditions. Cyclic stressing of a material may cause changes not only at that stress but at all other stresses.

Mathematical derivations appear in appendixes, and there are four discussions published at end of paper. Author's work represents first step towards experimental studies that may lead to new information on fundamental behavior of materials subjected to repeated stresses.

Alexander Yorgiadis, USA

2337. **Joseph Marin and D. E. Hardenbergh, Creep of laminates in tension, bending, and torsion,** Modern Plastics, 8 pp. (Feb. 1949).

Two materials, one a paper-base and the other a cotton-base laminate, are tested in tension, bending, and torsion. In addition to usual stress-strain curves, yield strength, ultimate strength, modulus of elasticity, and ductility are recorded. Materials are also tested in static tension, bending, and torsion creep by holding specimens at constant stress at about 77 F and at 50% relative humidity for a period of 1000 hr. Results are plotted on a log stress vs. log creep rate basis. Attempt is made to derive bending-creep rate theoretically, but results are not too satisfactory.

Edward Saibel, USA

2338. **Ladislav Boor, Some notes on the effect of testing machines on the tensile properties of plastic films,** Amer. Soc. Test. mat. Bull. no. 162, 47–53 (Dec. 1949).

Author presents load deformation curves for plastic and elastomeric films obtained on various types and makes of testing machines. He finds that load deflection characteristics for films are seriously affected by type of testing machine. Preferable type of tensile testing machine is one which utilizes a dynamometer system leaving negligible motion of grip.

F. J. Mehringer, USA

2339. **W. D. Jenkins and T. G. Digges, Influence of strain rate and temperature on the creep of cold-drawn ingot iron,** J. Res. Nat. Bur. Stands. 43, 117–125 (Aug. 1949).

Paper describes observations of several types of tests made on specimens of cold-drawn ingot iron. Tension tests were made at different strain rates and temperatures on specimens which were not carried to fracture in creep tests, and compared with specimens which had gone through a given thermal cycle only. From standard creep tests the relationship between true stress and minimum creep was established as not being linear on semi-log or double-log plots for investigated temperatures. Special creep tests were made in which load was reduced to maintain rate in third stage of creep at same value as that of second stage.

M. J. Manjoine, USA

2340. **W. G. Lidman and A. R. Bobrowsky, Correlation of physical properties of ceramic materials with resistance to fracture,** Nat. adv. Comm. Aero. tech. Note 1918, pp. 1–15 (July 1949).

Paper gives an analysis for determining properties of materials which affect resistance to fracture by thermal stresses set up by thermal shock. Thermal stresses induced during thermal shock are the result of temperature gradients from interior of material to surface. Resistance to fracture by thermal shock is shown to

be dependent upon thermal conductivity k, tensile strength s, thermal coefficient of expansion α, and ductility modulus E, (ratio of stress to strain at failure). Values representing these material properties are combined to give a single parameter, $ks/\alpha E$. Derived parameters are correlated with experiments that measure resistance of ceramic materials to fracture by thermal shock. Experimental work subjects a disk 2 in. in diameter and $^1/_4$ in. thick to a number of cycles of alternating heating and quenching until failure occurs. Center of specimen is heated from room temperature to an evaluation temperature (highest temperature of cycle, usually starting at 1800 F) in approximately six min and is held there for 10 min. It is then cooled from evaluation temperature to 400 F in about $1^1/_2$ min, and from 400 to 85 F in $4^1/_2$ minutes. When a specimen withstands 25 thermal shock cycles at a given evaluation temperature, test is continued at an evaluation temperature 200 F higher for 25 cycles or until failure, whichever occurs first. If no failure occurs, evaluation procedure is continued with next higher temperature.

Results of five ceramic materials are tabulated in paper and show good qualitative agreement between $ks/\alpha E$ parameters and experiment. Work is important for gas turbines in that it furnishes an index to suitability of various materials for resistance to failure by thermal shock. C. O. Dohrenwend, USA

2341. Wilhelm Bischof, Research on heat treatment of various alloy steels by isothermal reaction (in German), Arch. Eisenhüttenw. 20, 13–18 (1949).

Room-temperature impact tests of several series of alloy steels after heat treatment to hardnesses ranging from about 200 to 500 Vickers, by isothermal reaction and by quenching and tempering; 110-lb laboratory heats extending beyond ordinary composition ranges were used. Plain carbon heats from .27 to 1.07 C; chromium from 0.63 to 5.82 Cr at about .30 C; vanadium from .08 to 1.33 v; Mo from .25 to .92 Mo; tungsten .48 and .91 W, copper .44 and .84 cu, all at .40 C. S-curves for beginning and end of reaction. Ignores transition temperature. Below 300 Vickers, quenched and tempered tougher; above 400 vickers isothermally reacted tougher. Vanadium steels less tough after reacting and tempering at higher temperatures.

Maxwell Gensamer, USA

2342. C. J. Osborn, A. F. Scotchbrook, R. D. Stout, and B. G. Johnston, Effect of plastic strain and heat treatment, Weld. Res. Suppl. 14, 337–353 (Aug. 1949).

Paper is one of a series of reports describing results of work performed for the Pressure Vessel Research Committee of the Welding Research Council. Investigation describes effects of cold work and post-heating on properties of unwelded steel plates. These effects were evaluated by use of slow notched-bend and Charpy V-notch specimens. Results are contained in a series of tables and figures. George Gerard, USA

2343. Richard Weber, Properties and application of metallic slide bearing materials (in German), Zeit. Metallk. 39, 240–247 (Aug. 1948).

2344. Turner Alfrey, Jr., Mechanical behavior of high polymers, New York: Interscience Publishers, Inc., 1948, xiv + 581 pp. Cloth, 6 × 9 in., $7.75.

This clearly written and extremely interesting treatise is a successful effort to examine the mechanical properties of high polymers from a basic point of view. Author points out fundamental principles underlying mechanical behavior, and shows how this behavior is correlated with molecular structure. Attention is focused on those polymers whose mechanical properties have been studied from this point of view. Polymeric structures are classified as amorphous, linear polymers, cross-linked polymers, crystalline polymers, and polymers admixed with components of low molecular weight. Analysis of mechanical behavior is made on two levels, a phenomenological description and a molecular interpretation.

An idea of the scope of the work may be obtained from a partial listing of some of the topics treated. In the introduction the concepts of stress and strain are developed. This is followed by a brief introduction to the theory of elasticity, the mechanics of Newtonian liquids, nonNewtonian fluids, plasticity, and thixotropy. Next section deals with plastoelastic behavior of amorphous linear high polymers, and includes such topics as analysis of models of various combinations of Maxwell and Voigt units, and application to such phenomena as deformation-time curves, creep, effect of temperature, strain-rate, etc. Third section deals with three-dimensional cross-linked polymers and various theories of structure and their relations to mechanical properties, influence of molecular weight, etc. This is followed by a section on crystallization of high polymers, one on plasticization and solution, and, lastly, ultimate strength and related properties. Appendixes are devoted to tensor representation of stress and strain, mathematical methods of specifying viscoelastic properties, tensor treatment of nonhomogeneous stresses in viscoelastic media, and experimental methods in the scientific mechanical testing of high polymers. In each chapter author's system is to first analyze molecular structure and then to present phenomenological and mechanistic analyses together.

References are quite complete. This is the sort of book that should be read by all workers in the field of high polymers, regardless of special interest. Edward Saibel, USA

2345. R. D. Andrews, N. Hofman-Bang, and A. V. Tobolsky, Elastoviscous properties of polyisobutylene. I. Relaxation of stress in whole polymer of different molecular weights at elevated temperatures (abstracts in French and German), J. Polymer Sci. 3, 669–692 (1948).

Authors found a close relationship between stress-relaxation rate and molecular weight of various polyisobutylene specimens, indicating that relaxation tests may be used with considerable accuracy in determination of molecular weights of polymers directly in solid state. M. Hetényi, USA

2346. George C. Deutsch, Andrew J. Repko, and William G. Lidman, Elevated-temperature properties of several titanium carbide base ceramals, Nat. adv. Comm. Aero. tech. Note 1915, 52 pp. (1949).

With a view to establishing their usefulness in gas-turbine blade applications, elevated-temperature properties were measured for titanium-carbide ceramals with 5 to 30% of cobalt, molybdenum or tungsten added as a bonding metal. Tensile strength, modulus of rupture, coefficient of linear expansion and density were measured at temperatures ranging from 1600 to 2400 F. John E. Goldberg, USA

2347. H. Unkel, Deterioration tests with a copper-beryllium alloy (in German), Metall. no. 19/20, 319–324 (Oct. 1948).

2348. Henry L. Kennedy, Recent developments in concrete durability, J. Boston Soc. civ. Engrs. 34, 263–291 (Oct. 1947).

Extensive report on concrete durability, particularly as measured by freezing and thawing, and by resistance to chloride-salt action. Description of methods pertaining to utilization of air entraining agents, either added at the concrete mixer or interground with Portland cement. Thorough discussion of additive

materials (admixtures and addition agents) classified as follows: plasticizers, accelerators and catalysts, air-entraining agents, water-repelling agents, agents to cause controlled expansion, pozzuolanic materials, natural cements and retarders. Early history of durability due to air entrainment and effects of air-entraining agents on durability, consistency, strength, and other properties of concrete are described. Also included is discussion of methods of redesigning air-entrained concrete mixes and of measuring entrained air.

Author sums up beneficial effects of entrained air in Portland-cement concrete as follows: (1) Resistance of concrete to freezing and thawing and chloride salt action is greatly improved. (2) Workability of concrete in the plastic state is greatly improved, which results in easier placing, reduced segregation, and better finishing. (3) Green settlement, hence bleeding, is reduced with consequent improvement in ease and speed of finishing. (4) Permeability is reduced, due to greater density of concrete matrix surrounding microscopic air bubbles. (5) Resistance to disintegrating action of sulphate waters, alkali, and sea water is increased owing to greater impermeability of air-entrained concrete. Author also concludes: (1) Plasticity due to an air-entraining addition or admixture depends upon amount and grading of fine aggregate. (2) Optimum benefits are derived from air entrainment only in relatively lean mixes (less than $6^1/_2$ sacks), unless air content is held at a low figure, i.e., 2–3%. (3) Air entrainment beyond 6% by volume is not recommended. (4) For pavement, concrete air content within 2–4% improves the properties. (5) For wintertime construction, usual precautions for adequate curing and close control of air content below 4% are necessary in structural concrete. J. J. Polivka, USA

2349. **J. A. Sauer, A study of fatigue phenomena under combined stress,** Proc. seventh int. Congr. appl. Mech. 4, 150–164 (1948).

Bars of $^5/_8$-in. diam, made from aluminum alloy (14S-T) were tested under alternating stresses produced by pure torsion, pure bending, and combined torsion and bending. For pure torsion and bending, additional tests were made in which a mean stress other than zero was applied. Fatigue strength was based on cracking or failure at 10 million cycles.

Results of combined loading tests were plotted in form of an interaction curve by using the ratios between principal stresses and fatigue strength for pure bending. The points were close to straight line, represented by equation: $(\sigma_1/\sigma_r) + (\sigma_2/\sigma_r) = 1$, where σ_1 and σ_2 are principal stresses, and σ_r is fatigue strength in pure bending.

This line corresponds to maximum shear stress theory of failure and gives values lower than maximum stress or maximum distortion energy theories of failure. It appears to be a good basis for design of members subjected to repeated combined stresses.

Superposition of a static (constant) stress onto an alternating stress state was found, as usual, to decrease fatigue strength (when latter is expressed in terms of alternating stress only). For pure torsion the reduction was negligible up to the point where maximum stress reached 60 per cent of static yield stress. For pure bending a linear reduction was found, but the amount of reduction appeared to be less than that given by straight-line or parabolic formulas. F. R. Shanley, USA

2350. **K. Fink, On impact tests with samples of mild steel and some general conclusions for solid materials under impact load** (in German), Schweiz. Arch. 15, 193–214 (July 1949).

In specially designed test equipment, mild-steel test specimens were subjected to tensile or compressive impact loads. Specimens were attached to dynamometer rods provided with carbon-strip resistance strain gages for measuring the time history of the force. Test results for four steels, three of which had a pronounced upper yield point, and cold-drawn copper are given (with many photographs of force-time curves) and explained by means of Neumann's theory of elastic wave propagation.

Steel with 0.02% C having a pronounced upper yield point showed increase of yield stress under impact loads of more than 100% over the static yield stress, when former was reached in 10^{-4} sec. For higher carbon contents, relative increase of yield stress diminished.

It is emphasized that recourse should be had to considerations of wave propagation even for normal impact tests. In a final section, a standard procedure for impact testing of materials (central longitudinal impact) is proposed.

F. J. Plantema, Holland

2351. **Method of test for determining strength properties of core material for sandwich construction at normal temperatures,** For. Prod. Lab. Rep. 1555, 11 pp. (March 1948).

Described tests are: (a) Compression and measurement of Young's modulus (flatwise and edgewise); (b) tension (flatwise); (c) determination of modulus of rigidity G by panel shear test; (d) determination of G by torsion pendulum; (e) shear strength test and measurement of G; (f) determination of moisture content and specific gravity. For tests (b) and (c) material is stuck to load-transmitting blocks and results are valid only when no glue-line failure occurs. Suitable adhesives are described, and methods of constructing and testing specimens are explained and illustrated in detail. Control of humidity and temperature is recommended except for moisture-sensitive materials which should be tested dry.

Reviewer finds tests fair and practical. However, in test (e), as in every lap-joint type of shear test, failure is usually initiated not by a shear but by high tension developing at edges of overlap. Special steel end plates covering edges probably complicate stress distribution even more. Pure uniform shear may be produced by torsion of a ring of material glued between bases of two aligned tubes. C. Mylonas, England

2352. **D. N. Frey, J. W. Freeman, and A. E. White, Fundamental effects of aging on creep properties of solution-treated low-carbon N-155 alloy,** Nat. adv. Comm. Aero. tech. Note 1940, 73 pp. (Aug. 1949).

N-155 material is an alloy containing about 20% each of chromium, nickel, and cobalt with a few per cent each of manganese, molybdenum, tungsten, and columbium. Solution treatment which preceded aging treatment was 10 hr at 2200 F and water-quenched. Treatment was made unusually long to remove all traces of former working and to distribute precipitant atoms randomly throughout matrix.

Aging treatments were carried out at 1200, 1400 and 1600 F for time periods of 1, 10, 100 and 1000 hr. Optical and electron micrographs were prepared to study progress of precipitation in grain boundaries. X-ray studies were made to investigate diffraction-line peak intensities and diffraction-line broadening. Lattice parameter studies also were made. Hardness at end of each aging period was determined on a Brinell machine. Creep and creep rupture tests were run at 1200 F. Duration of tests was limited to 50 hr in order that testing time would be considerably less than longest of aging times.

Results of these studies are reviewed thoroughly. Among results and conclusions the most important are indicated by three incomplete quotations: (1) "Aging of solution-treated low-carbon N-155 resulted in progressive lowering of initial (short-time) creep resistance through removal from solid solution of large-

radius or interstitial atoms by precipitation. . ." (2) "Short-time aging of solution-treated low-carbon N-155 apparently resulted in a marked increase in short-time rupture strengths over that of unaged material through growth of a grain boundary phase, this phase acting to strengthen boundary areas to eliminate intergranular cracking. . . ." (3) "Because effect of aging in general was to lower creep resistance and to raise rupture strength, for time periods considered, aging resulted in a material which exhibited greater ductility before fracture." E. A. Davis, USA

2353. F. V. Warnock and D. B. C. Taylor, The yield phenomena of a medium carbon steel under dynamic loading, Inst. mech. Eng. appl. Mech. Proc. 161, 165–175 (1949).

Paper is concerned with permanent strain in tensile specimens of medium carbon steel which are loaded at rapid strain rates. To overcome difficulties involved in measuring continuous rapid stress and strain, authors subject specimen to a series of repeated "shock" loadings and measure permanent strain after each loading.

Purpose of tests is twofold: (1) To study effect of rate of straining upon the permanent strain and in particular to compare results of dynamic loading (maximum strain rates from 7.7 to 12.9 in. per in. per sec) with static loading; and (2) to compare permanent strain in "normalized" specimens, from which internal stresses have been removed by a heat treatment, with "as received" specimens which contain uneven internal stresses. It is generally found that subjecting specimen to dynamic loading produces same qualitative changes in its permanent strain and in a greater degree, as does normalizing it. These effects are: (1) Considerable increase in upper yield stress; (2) lesser increase in lower yield stress; (3) increase in permanent strain before strain hardening takes effect; and (4) greater variation of permanent strain over length of specimen. For all types of testing considered, variation over length is most marked after one shock loading and tends to become more nearly uniform as number of loadings is increased.

Testing apparatus is described in some detail. Care has been taken to avoid reaction-stress waves and insure that loading is purely tensile. Presentation of results has been facilitated by use of photographs, figures, and graphs. In particular, about 15 graphs appear to be well planned to facilitate the comparison of results.

Paper concludes with a tentative explanation of differences between dynamic and static loading and between "normalized" and "as received" steels, in terms of elastic-plastic yield theory. While discussion appears reasonable, it is quite brief and of a qualitative nature. Primary value of paper definitely lies in its well-conceived experiments and well-presented experimental results.

Philip G. Hodge, Jr., USA

2354. H. Krainer, K. Swoboda, and F. Rapatz, Low-alloyed heat resistant steels (in German), Arch. Eisenhüttenw. 20, no. 3–4, 111–114 (Mar.-Apr. 1949).

Authors investigated suitability of Cr, V, Mn, and Si as substitutes for W and Mo in hotwork tool steels. For this purpose 40 steels with varying compositions were tested for their temper resistancy, tensile strength, hot impact value, heat conductivity, and firecrack resistancy. It was found that V improves temper resistancy, provided that at higher V-contents (>0.4%) Cr content is 1.5%. Mo improves deep hardening characteristics and hot tensile strength. W improves temper resistancy and hot tensile strength and especially decreases sensitivity to fire cracking. Most favorable properties on application of smallest amounts of W and Mo were obtained with a steel containing 1.5% Cr, 0.9% V, 0.5% Mo and 0.5% W. This steel proved to be very suitable for hot pressing dies and casting dies. J. H. Palm, Holland

2355. H. B. Saldin and P. G. Dehuff, Jr., Hot-spin tests of bladed jet-engine rotors, Trans. Amer. Soc. mech. Engrs. 71, 605–612 (1949).

Authors describe: (1) Design and construction of a test pit with auxiliary apparatus suitable for testing performance of high-temperature rotating parts in accordance with a predetermined schedule of temperature gradient, temperature, and speed; (2) testing procedure; and (3) results of tests on four-bladed disks.

The disks were made respectively of: (*a*) 19-9-DL material, hot-cold worked, stress relieved at 1200 F, conforming to AMS-5521 (Bhn 229–255); (*b*) same as (*a*) but with Bhn 285–311; (*c*) 16-25-6 Timken alloy, solution treated and cold-forged to Bhn 240 to 300; and (*d*) G18B hot-cold worked to Bhn 235–255.

Standard 19-9-DL showed best combination of strength and ductility, although the Timken 16-25-6 alloy as processed surpassed it somewhat in strength. Cold-worked 19-9-DL had least desirable properties. Work is being continued in search for strength and ductility design criteria for materials used in high-temperature rotating parts. Hans F. Winterkorn, USA

2356. James J. Gangler, Chester F. Robards, and James E. McNutt, Physical properties at elevated temperature of seven hot-pressed ceramics, Nat. adv. Comm. Aero. tech. Note 1911, 36 pp. (July 1949).

To evaluate their applicability for use in gas turbines, seven hot-pressed ceramics were tested at elevated temperatures to measure their short-time tensile strength, thermal-shock resistance, coefficient of linear expansion, and density. Of tested materials, titanium carbide appears to be most promising, having best resistance to thermal shock and reasonably good tensile strength of 9400 psi at 2200 F. Hot-pressing technique, an adaptation of a process used in powder metallurgy, makes possible production of high-density ceramics. John E. Goldberg, USA

Mechanics of Forming and Cutting

(See also Revs. 2314, 2342)

2357. H. I. Ansoff, Forming of a plastic sheet between fixed cylindrical guides with Coulomb friction, Trans. Amer. Soc. mech. Engrs. 72, 145–152 (1950).

Problem considered is drawing a plastic sheet between two fixed cylindrical surfaces under assumption that material between cylinders is in a state of plane plastic flow and behaves according to Saint Venant-Mises theory. General equations of plane plastic flow are discussed and their reduction to a system of quasilinear partial differential equations given. Boundary conditions, including that of Coulomb friction along the cylindrical surface, are given. Equations are solved by method of characteristics. Three basic regions in the sheet must be considered in the solution. The first turns out to be a "region of constant stress," the second represents a "noncentered fan," and the third region requires a numerical solution. Two methods are given for this region: Approximation by a system of linear difference equations, and construction of characteristic curves by using approximate radii of curvature. Latter method was suggested by W. Prager [Courant Anniv. Vol., pp. 289–300, New York, Interscience (1948); Rev 1, 186]. Results show good agreement with those obtained by previous writers. Author also shows that in the practically important cases of small sheet thickness to cylinder radius ratios, solution can be obtained by a simple approximation. Results, thus obtained, agree extremely well with those of Sachs [G. Sachs and L. J. Klinger, J. appl. Mech. 14, A-88–A-98 (1947)]. Paper concludes with discussion of determination of elastic-plastic boundary.

G. H. Handelman, USA

2358. **E. H. Lee and B. W. Shaffer, The theory of plasticity applied to a problem of machining,** Graduate Div. of applied Math., Brown University, Providence, R. I., tech. Rep. A11–43, ii + 57 pp. (1949).

Authors obtain analytical expressions relevant to problems of orthogonal machining. Account is taken of effect of stress distribution between work material in plastic flow and tool face, under assumption that material is ideally plastic and does not exhibit work hardening. Comparisons are made with other theories, which do not explicitly consider mode of force transmission into work material, and importance of this consideration is verified by reference to experimental data.

Courtesy Mathematical Reviews F. B. Hildebrand, USA

2359. **H. H. Miller, New cut-wire shot big boon to peening,** Soc. auto. Engrs. J. 57, 44–51 (1949).

Peening automotive chassis springs with cut-wire shot instead of chilled cast-iron shot improves quality and lowers costs. Peening with cast-iron shot has been a costly and uncontrollable operation because of excessive peening usage, short shot life, lack of size uniformity and quality of shot as furnished, and formation of grits which makes necessary frequent overhaul of peening equipment. By using cut-wire shot made by cutting hard-drawn mechanical spring wire of 0.0348 in.-diam with a tensile strength of 261,000 to 301,000 psi, these drawbacks were overcome.

Cut-wire shot has a lower hardness than the chilled cast-iron shot and this, in connection with a lesser shot usage, is reason why total cost of shot plus maintenance of peening equipment is, per spring, about 50% lower than cost when cast-iron shot is used. Another essential advantage is increased fatigue-life of coil spring peened with cut-wire shot. Other machine parts, e.g., inside surface of Dynaflow-brake bands, are now also peened with cut-wire shot.

Article is accompanied with many photos showing shape and surface of peening particles and surface of peened coil springs.

Ferdinand Budinský, Czechoslovakia

2360. **W. Lueg, Determination of increase in width for hot-rolling of steel by smooth rolls** (in German), Arch. Eisenhüttenw. 20, 59–68 (Jan.-Feb. 1949).

Author reviews several expressions which have been proposed by various investigators for determination of increase in width for hot-rolling of steel under smooth rolls. Some formulas are empirical and some are based on analysis of deformation produced by rolls. Formulas by A. Falk (1910), W. Tafel and H. Sedlaczek (1925), S. Ekelund (1927), F. Riedel (1936), E. Siebel (1937), J. Roux (1939), and N. Marini (1941) are compared. If b is width of a piece after rolling, b_o before it enters the rolls, h_1 its thickness after, and h_o before rolling, most of formulas express $\Delta b = b - b_o$ as a function of decrease in thickness $\Delta h = h_o - h_1$ and of thickness-ratio h_o/r, r denoting radius of the rolls; some of formulas also contain coefficient of friction μ between rolls and piece to be rolled, and yield point of steel. Author compares these formulas by plotting ratios of increase in width $\Delta b/\Delta h$ as a function of ratios of width to thickness b_o/h_o. All expressions, with exception of one, agree in general trend of curves; curves show an increase of $\Delta b/\Delta h$ with increasing values of b_o/h_o to a maximum and a subsequent decrease of $\Delta b/\Delta h$ at large values of b_o/h_o. Author discusses merits of these formulas and presents them in a number of monograms.

A. Nadai, USA

2361. **Michael Field and M. E. Merchant, Mechanics of formation of discontinuous chip in metal cutting,** Trans. Amer. Soc. mech. Engrs. 71, 421–430 (July 1949).

Using previous work on mechanics of formation of continuous type of chip as a background, authors develop an analysis of main features of geometry of chip formation and mechanics of cutting where a segmental or discontinuous type of chip is formed. In such chip formation, segments are formed by rupture which occurs intermittently on shear plane as cutting tool progresses. Geometrical and force relationships, plasticity conditions, and conditions controlling rupture on shear plane are analyzed. An experimental study of process, particularly of rupture conditions, is included. Rupture is shown to be correlated with shearing strain and compressive stress on shear plane. As tool advances into cut immediately following rupture of a segment, metal is first deformed plastically, and angle of shear is large. Rupture is prevented during this stage by the fact that compressive stress on shear plane is high enough to permit metal to undergo considerable shearing strain without fracture. As tool progresses, shear angle falls until compressive stress becomes sufficiently low (or shearing strain becomes sufficiently high) to permit fracture.

M. E. Merchant, USA

Hydraulics; Cavitation; Transport

(See also Revs. 2382, 2390, 2551)

2362. **André Jorissen, Contribution to the study of continuous losses of head in circular conduits** (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale 4, 237–319 (1949).

Of the many papers during past several decades on the subject of pipe flow, by far the majority have either summarized analyses of others without making any further contribution, or have presented original data which are too restricted or too scattered to do more than becloud the issue. Paper is a refreshing departure from such practice, in that it contributes perceptibly to both analysis and measurement of pipe flow. The first half of treatment represents a critical review of existing formulas and figures, with emphasis on a reappraisal of Nikuradse data after elimination of an adjustment factor inexplicably used in original paper. Also discussed, among others, are transition functions of Colebrook and Galavics.

Second half is a presentation of resistance data on new steel pipe 60, 80, 100, and 150 mm in diam over a Reynolds-number range from 2000 to 800,000. When presented in customary manner, data show much the usual scatter expected from measurements on commercial pipe, general trend being about midway between Colebrook and Galavics transition functions. Of special interest, however, is author's use of a pneumatic sensing device for direct measurement of absolute roughness magnitude, which was found to vary appreciably from section to section and from pipe to pipe. Resistance measurements correlated in terms of this independent parameter are shown by author to have sufficient precision to warrant practical use of method. [Reference should be made to author's discussion of Paper 1296 in Proc. Amer. Soc. civ. Eng. 75, 1533–1537 (Dec. 1949).] Hunter Rouse, USA

2363. **Ettore Scimemi, Experimental results on the propagation of regimen-perturbations in open channels** (in Italian), Energia Elett. 26, 691–696 (Nov.-Dec. 1949).

Study was made of effect of sudden reduction in flow on canal feeding the penstocks of Cismon-Moline hydroelectric plant in province of Venice in northern Italy. Section of studied canal, about 1100 meters long, fed directly into penstock, was in a tunnel with a bottom slope of $1^1/_2\%$, and was preceded by a short reach of tunnel with an 8% slope. In this reach, water surface was almost horizontal. Capacity of tunnel was 17 m^3/sec. One full-scale test was made of actual installation in which flow was reduced suddenly from 13.5 m^3/sec to zero. Observations were made of water-surface elevation as a function of time at several

stations in this reach. Problem was then taken into hydraulics laboratory of University of Padua where model was made of the reach of tunnel with a linear ratio of 1:25 and same test was repeated. Results of this model test and of prototype tests agree very closely. A number of other tests were made in the model with different flow rates. Problem was also analyzed using a method developed by P. Forchheimer. Results of analysis agree closely with those obtained by experiment.

Vito A. Vanoni, USA

2364. Francesco Ramponi, On the forms of the outfall of channels and of works with superficial discharge (in Italian), Energia Elett. 26, 453–460 (Aug. 1949).

Effect of variations in shape of entrances into canals was studied by means of models constructed in hydraulics laboratory of University of Padua. Effect upon discharge of rounding crest and sides of entrance to different degrees was investigated, and variation in contraction of stream noted for various conditions. Widths of models were of order of 8 in. Results are presented in dimensionless graphs. Glenn Murphy, USA

2365. J. A. Putnam, Loss of wave energy due to percolation in a permeable sea bottom, Trans. Amer. geophys. Un. 30, 349–356 (June 1949).

A theory is developed for loss in wave energy as a result of currents induced in a permeable sea bottom by wave action. This loss in energy is reflected as a reduction in wave height as wave moves shoreward. Numerical examples show that this reduction may be on the order of 10% on very flat beaches for wave periods and bottom materials commonly found in the ocean.

From author's summary by Walter H. Munk, USA

2366. A. J. Stepanoff, Elements of graphical solution of water-hammer problems in centrifugal-pump systems, Trans. Amer. Soc. mech. Engrs. 71, 515–534 (July 1949).

Since publication of "Symposium on Water Hammer" in 1933, considerable progress has been made abroad in development of graphical method of solution of water-hammer problems. Although based upon same fundamental relationships as analytical solutions, graphical method does not require an exact knowledge of theoretical background and, at same time, it gives a clear mental pattern of events which enables one to use it with confidence for solution of practical problems.

Early studies of water-hammer problems were carried out almost entirely in connection with water-turbine applications. Water-hammer problems in centrifugal-pump field are more numerous and varied, but owing to the smaller size of the units, effects of water-hammer were learned by experience, damage being repaired at a relatively low cost. Centrifugal-pump designers and users are seldom familiar with water-hammer theory, largely due to fear of involved mathematical treatment. Author attempts to present water-hammer theory and the graphical method of solution in its simplest terms. This may help a pump engineer to recognize conditions leading to water hammer, to estimate possible maximum pressure rise and, when necessary, to provide means for reducing pressure rise to a safe limit.

R. C. Binder, USA

2367. R. S. Garthune, B. Rosenberg, B. Cafiero, and C. R. Olson, The performance of model ships in restricted channels in relation to the design of a ship canal, David Taylor Model Basin Report no. 601, 229 pp. (Aug. 1948).

Results are reported on a series of tests made with models of several representative types of ships operating in restricted canals. Tests were made to obtain information to guide selection of cross sections and bends for a modernized canal across Isthmus of Panama. Because experiments cover a relatively wide range of variables, they should be of general use to designers and operators of ship canals.

Three types of tests were made. First type involved a single ship maneuvering in a restricted canal. Principal phenomenon investigated here is so-called bank suction, or tendency of ship to be drawn towards near bank. Second type involved study of interaction between ships passing in opposite directions in a canal. Third type involved measurements of sinkage or distance that ships settle in water when they are set in motion.

Ship models used in tests represented a large ship, a class of ships which are known to have poor handling characteristics in restricted canals, and a type which is representative of a large fraction of ships using the Panama Canal. Linear scale ratios of models were 1 to 35 and 1 to 45. Ship models were large, being between 10 and 20 ft in length. Variables studied included water depth, channel width, ship speed, distance of ship from channel centerline, and velocity of currents in canal.

Tests with a single ship in a canal were made with self-propelled free-running models and with stationary models with water in motion. With free-running models, a rudder position was determined which kept ship on a straight course. With fixed models, actual forces on ship were measured as functions of yaw, rudder angle, position of ship in channel, and water velocity.

In studies of interaction between two ships meeting in a canal, one of the models was towed in a straight line and the other was self-propelled and free-running and was steered by an experienced Panama Canal pilot. Experienced pilots also steered free-running single models in straight and curved canals. Sinkage measurements were made on free-running models.

Vito A. Vanoni, USA

2368. A. P. Yufin, Motion of a heterogeneous liquid in horizontal non-silted steel pipes (in Russian), Izvestiya, U-Ser. tekh. Nauk no. 8, 1146–1159 (Aug. 1949).

Results are reported on experimental pumping of sand-water mixtures through a system of pipes connected in series and having diameters of 450, 350, 300, and 250 mm. Sands of five different grain sizes, ranging from 0.25 to 7.36 mm, were used. Results obtained agreed with those of O'Brien only at velocities higher than velocity which is critical in respect to settling out of particles. For lower velocities, a coefficient correcting conventionally computed gradient losses for difference between discharge density and actual density was determined as a function of pipe diameter, size of solid particles, consistency of mixture, and its average velocity. It is suggested that subcritical velocities may be more economical for field use. Note typographical error on fig. 8: in lower half of diagram for d read D. This diagram summarizes results of supplementary tests on critical velocities.

Gregory P. Tschebotarioff, USA

2369. A. M. Binnie and J. F. Davidson, The flow under gravity of a swirling liquid through an orifice-plate, Proc. roy. Soc. Lond. Ser. A 199, 443–456 (Dec. 1949).

Paper deals with flow through circular sharp-edged hole in bottom of a tank, when conditions are fulfilled for establishment of an irrotational whirlpool. Shapes of free surfaces of both whirlpool and emergent jet, and velocity distribution in a meridian plane are numerically determined for a particular case. Solution is by means of relaxation methods. Results of the calculations are compared with experiments made with water. The stability conditions for the annular swirling jet are analytically examined, taking into account only the centrifugal force and the surface tension (neglecting the effect of gravity). For cases of in-

stability, the preferential wave length of the vibrations is also established (about 4, 5 times the external diameter of jet), which is found in rough agreement with the experimental results.

Duilio Citrini, Italy

Incompressible Flow: Laminar; Viscous

(See also Revs. 2429, 2436, 2437, 2458, 2461)

2370. **Giulio Supino, On the irrotational motion of viscous liquids** (in Italian), R. C. Accad. Lincei 6, 615–620 (1949).

Author proves three theorems for viscous liquids: (1) A viscous liquid cannot move irrotationally if the field of flow is bounded by a part, however small, of a rigid wall at rest or in uniform translation. (2) If the field is bounded by rigid walls in helicordal or rotatory motion, the search for irrotational motions leads to conditions which are, in general, incompatible. It cannot be asserted that they are always incompatible for there are only two types of irrotational two-dimensional motion. (3) In the case of a liquid bounded only by free surfaces there are no irrotational motions, if the region occupied by the liquid is finite and simply connected. In case of a canal, there are no irrotational waves of permanent type, even if canal is of infinite depth and breadth.

L. M. Milne-Thomson, England

2371. **Susumu Tomotika and Ziro Hasimoto, Studies of discontinuous flow, I. The discontinuous flow past a flat plate placed near a plane wall** (in Japanese), Studies in mathematical Physics, Iwanami Shoten Publ. Tokyo 1, 1–21 (1950).

The discontinuous flow of an incompressible perfect fluid past a flat plate placed near a plane wall is treated with aid of Villat's method, assuming a dead water region of Helmholtz-Kirchhoff's type behind the plate. Exact formula giving resultant fluid pressure on plate is obtained in terms of elliptic functions. Detailed numerical discussions are then given of the variation of C_R/C_{RO} (ratio of actual drag coefficient to that pertaining to the unbounded stream) as a function of l/H (ratio of plate length to distance from wall). It is found that C_R/C_{RO} increases or decreases approximately linearly with l/H according as angle of attack β is smaller or larger than 90 deg, while for $\beta = 90$ deg, C_R/C_{RO} remains identically equal to 1, so that wall effect on drag does not exist in this special case.

Isao Imai, Japan

2372. **Susumu Tomotika and Ziro Hasimoto, Studies of discontinuous flow, III. The discontinuous flow past a flat plate which touches a plane wall with its trailing edge** (in Japanese), Studies in mathematical Physics, Iwanami Shoten Publ. Tokyo 1, 43–55 (1950).

Drag and moment acting on a flat plate which touches a plane wall are considered, and detailed numerical discussions are given of their values as functions of angle of attack β. It is found that drag becomes maximum at $\beta = 81$ deg, and that center of pressure moves rearward from midpoint of plate as β increases from 0. Free streamlines are also shown graphically for several values of β. It is pointed out that the wider the breadth of wake, the larger the drag.

Isao Imai, Japan

2373. **Susumu Tomotika and Ziro Hasimoto, Studies of discontinuous flow, II. Bobyleff's problem** (in Japanese), Studies in mathematical Physics, Iwanami Shoten Publ. Tokyo 1, 22–42 (1950).

Authors show that results obtained previously (see preceding review) for discontinuous flow past a flat plate in presence of a plane wall are reduced to known results for Bobyleff's problem in limiting case when trailing edge touches wall. (Bobyleff's problem is concerned with flow impinging symmetrically on a bent lamina which consists of two equal flat plates forming an angle.) Analysis is based on use of Jacobi's imaginary transformation of theta functions.

Isao Imai, Japan

2374. **H. B. Helmbold, On the calculation of profile resistance** (in German), Ingen.-Arch. 17, no. 4, 273–279 (1949).

Paper begins with a summary of boundary-layer theory for a profile or body of revolution in an incompressible fluid. Effect of a pressure gradient along body surface is considered and shape parameter in case of turbulent flow is introduced. Results for laminar flow are also developed, as well as for combined case of laminar and turbulent flow. Equations for profile drag coefficient are finally given, but no numerical results or comparison with experimental data are presented.

M. J. Thompson, USA

2375. **J. S. McNown, H. M. Lee, M. B. McPherson, S. M. Engez, Influence of boundary proximity on the drag of spheres,** Proc. seventh int. Congr. appl. Mech. 2, part I, 17–29 (1948).

An investigation is made by authors to establish influence of boundary proximity on drag force which is exerted upon a sphere moving at constant velocity along axis of a cylinder. Study is carried out for nearly all values of ratio of sphere diameter to diameter of enclosing cylinder (d/D), and for a range of Reynolds numbers of sphere from about 10^{-2} to 10^3.

For limiting case of $d/D = 0$, and with no inertial effects, Stokes law for drag on a sphere in a fluid of infinite extent is obtained. As has been found previously, this holds for Reynolds numbers below about 0.2, inertial effects being important above this. For Reynolds numbers below 0.2, authors find from experiment that the correction factor which must be applied to Stokes law increases with increasing d/D as prescribed by theory [H. M. Lee, MS thesis, State Univ. of Iowa (1947)] for d/D less than 0.25; this correction factor follows a second theoretical expression for d/D greater than 0.95, provided inertial effects are small.

Experiments have been used to establish relation of this correction factor to values of d/D throughout entire range, and have made possible the construction of a graph of drag coefficient versus Reynolds number for different values of d/D. Latter graph shows that critical limit of Reynolds number below which inertial terms are unimportant increases with increasing d/D (from 0.2 at $d/D = 0$ to 70 at $d/D = 0.8$).

Authors indicate possibility of using results of study for purpose of establishing a rational boundary and concentration correction factor for determination of sediment size by settling velocity technique.

Reviewer found paper rather incomplete and consequently somewhat difficult to follow, both with respect to definition of terms and in regard to explanation of manner in which experimental data was processed and final curves constructed. Despite this, investigation leaves little doubt of importance of influence caused by boundaries adjacent to a freely falling sphere, and indicates necessity of further investigation, especially for practical problem of sediment size analysis.

Robert O. Reid, USA

2376. **C. A. Truesdell and R. C. Prim, Vorticity and the thermodynamic state in the flow of an inviscid fluid,** Nav. Ord. Lab. Mem. 9416, 14 pp. (1947).

Paper develops relations connecting vorticity and thermodynamic variables in flow of inviscid fluids. Steady flow of a perfect gas is considered more in detail. Convenient forms of relations are given from which simple conclusions can be drawn.

C. C. Lin, USA

2377. C. A. Truesdell, A new definition of a fluid, Nav. Ord. Lab. Mem. 9487, 31 pp. (1948).

Author's purpose is to establish the most general form of stress tensor as a power series in the viscosity coefficient. First, through dimensional argument, he introduces three reference quantities, reference viscosity, reference temperature, and a quantity which has the dimension of the gas constant in equation of state for a perfect gas. Dissipation function is then considered to be a function of these three reference quantities and mean pressure, thermodynamic pressure, temperature, deformation tensor, and vorticity; derivatives of pressure, temperature, and external force field; and molar concentrations of components. Final form of stress tensor is derived on condition of isotropy and complete tensorial and dimensional invariance.

It seems to reviewer that (1) the equations of motion used by author are Eckart's equations which are probably incorrect [see Hirschfelder and Curtiss, J. chem. Phys. 17, 1076–1081 (1949)]; (2) author's doubt on Burnett's equations derived from kinetic theory is unfounded and arises from neglect of pressure gradient in a pure rotation. H. S. Tsien, USA

2378. C. A. Truesdell, A new definition of a fluid, I. The Stokesian fluid, Nav. Res. Lab. Rep. P-3457, iv + 11 pp. (1949).

In a more restrictive treatment than another paper by author (see preceding review), a Stokesian compressible fluid is defined as a fluid whose dissipation function is only a function of reference viscosity and reference temperature, mean pressure, thermodynamic pressure, temperature, and deformation tensor.

H. S. Tsien, USA

2379. C. A. Truesdell, A new definition of a fluid, I. The Stokesian fluid, Proc. seventh int. Congr. appl. Mech. 2, part II, 351–364 (1948).

Aside from two remarks on theory of elasticity and the Reiner-Rivlin theory of fluids with relaxation, paper contains same material as in author's other two papers (see two preceding reviews).

H. S. Tsien, USA

2380. Paul Painleve, Resistance of non-viscous fluids. Edited by A. Métral and R. Mazet. II, revised by R. Mazet (Leçons sur la resistance des fluides non-visqueux) (in French), Paris, Serv. de Docum. et d'Inform. Techni. de l'Aéro. 1949, v + 210 pp. Price 1000 fr.

[Volume I was published by Gauthier-Villars, Paris (1930).] This volume is concerned with movement of cylinders and three-dimensional bodies in an unlimited region of ideal incompressible fluid. Joukowski hypothesis regarding circulation about a cylinder is discussed in detail. Witosznyski's and other attempts to avoid certain philosophical difficulties of Joukowski theory are discussed. Chapters on forces on three-dimensional bodies in accelerated motion follow classical lines. Theories of three-dimensional wings, such as Prandtl's, are not included. Book closes with a chapter, contributed by Métral, on motion of a particle under influence of gravity and a force proportional to speed squared. There are 28 pages of typographical errata appended to book. W. R. Sears, USA

2381. F. H. van den Dungen, On the application of the variational principle in fluid mechanics (in French), Colloques Internationaux du Centre National de la Recherche Scientifique, no. 14, Methodes de calcul dans des problèmes de mecanique, pp. 88–95. Centre National de la Recherche Scientifique, Paris, 1949.

Paper is a contribution to classical problem of deriving equations of motion of inviscid fluids from a variational principle, first partially solved by Clebsch, whose result was extended by Bateman [Proc. roy. Soc. Lond. Ser. A 125, 598–618 (1929); cf. also Physical Rev. (2) 38, 815–819 (1931)], analysis being confined to case of barotropic motion. Making use of two remarks of Bateman, author obtains a variational principle valid in baroclinic motion; the associated invariant integral appears as the Bjerknes circulation theorem. Only momentum equation is derived; variational principle recently obtained by A. H. Taub [Proc. Symp. appl. Math. 1, 148–157 (1949); Rev 3, 1316] yields the energy equation as well, but is restricted to the case when heat conduction is neglected. Author discusses also the case of infinitesimal oscillations. In Bouligand's comments on author's results, reference is made to the corresponding problem for viscous fluids [C. B. Millikan, Philos. Mag. (7) 7, 641–662 (1929)], recently reopened by H. Villat [Leçons sur les fluides visqueux, Gauthier-Villars, Paris (1943), chap. III] and solved by R. Miche [J. Math. pures appl. (9) 28, 151–179 (1949); Rev 3, 914].

C. A. Truesdell, USA

2382. Fukusaburo Numachi and Tatsuro Yokoyama, Cavitation tests on hydrofoil profiles suitable for arrangement in a straight grate (in Japanese), Rep. Inst. high speed Mechanics, Tohoku Univ. 1, 89–99, 107–119; 2, 25–46 (1949).

Measurements were made in cavitation tunnel on several airfoils which were designed to have pressure distribution similar to that of laminar-flow airfoil. Their characteristics were found to be excellent as compared with conventional airfoil, especially at small values of cavitation coefficient. Itiro Tani, Japan

2383. Fukusaburo Numachi, Cavitation tank for hydrofoil-grate and its theoretical basis of experiment (in Japanese), Rep. Inst. high speed Mechanics, Tohoku Univ. 1, 45–56 (1949); **Fukusaburo Numachi, Sadatoshi Fuchizawa, and Kenji Tsunoda, Cavitation tests on hydrofoils arranged in a straight grate** (in Japanese), Rep. Inst. high speed Mechanics, Tohoku Univ. 1, 57–75 (Aug. 1949).

Lift and drag were measured in water tunnel on airfoils arranged in a cascade for cavitation coefficient k_d from 0.4 to 2.5. Airfoils were Clark Y, Clark YH and ogival form, thickness ratio being 6%. Visual observations were also made on inception of cavitation. Upper and lower walls of tunnel have been built to be adjustable so that flow passed through cascade assumes direction required from theoretical ideal flow through infinite cascade.

Itiro Tani, Japan

2384. N. F. Barber, The behaviour of waves on tidal streams, Proc. roy. Soc. Lond. Ser. A 198, 81–93 (1949).

Ocean waves reaching west coast of England from south Atlantic storms show frequency modulation correlated with semi-diurnal tides. This is explained as effect of currents and depth fluctuations over continental shelf. Treatment is based on an undemonstrated hypothesis that there can exist in this case elementary wave trains in which individual crests persist and obey usual relation between velocity, wave length, depth, and gravity, in spite of gradual changes in depth, horizontal flow, and effective gravity, as functions both of time and position. Effects of these variations on wave length and period are derived; in general, the partial derivatives with respect to position determine wave-length changes, and those with respect to time, period changes. Agreement with observation is satisfactory. Philip Rudnick, USA

2385. A. M. Faĭnzil'ber, Thermal loss of energy in flows of a viscous fluid at large Reynolds numbers (in Russian), Doklady Akad. Nauk SSSR 69, 503–506 (1949).

Author starts with differential equations of a viscous fluid for

large Reynolds numbers. Into these he introduces the dimensionless quantities $y_1 = [\bar{u}(x) - u(x, y)]/\bar{u}(x)$, $x_1 = x/x_m$ as independent variables and the relative loss of energy $E_1 = 2\mu(\partial u/\partial y)^2/(\rho \bar{u}_k^2)$ as the dependent dimensionless variable. The equation for E_1 is then solved in the series form $E_1 = \sum_1^\infty B_i (x_1)_{i_1}^{(i)}$ (m and k are some characteristic points; for example, m may be taken as the point of minimum pressure; both points may coincide). The new variables are found to be suitable for both finite and asymptotic layers. Distribution of velocities and distribution of dimensionless temperature are described using known E_1. Finally author considers special case when the free-stream velocity distribution is prescribed to be $\bar{u}(x_1) = (\alpha x_1 + \beta)^r$ (power distribution of velocities). E. Leimanis, Canada

2386. **R. S. Rivlin, The normal-stress coefficient in solutions of macro-molecules,** Trans. Faraday Soc. 45, 739–748 (Aug. 1949).

Concept of a nonNewtonian fluid has been introduced by author [Proc. roy. Soc. A 193, 260 (1948); Rev 2, 77]. In present paper type of flow considered is such that fluid velocity at point (x, y, z) is (K_1x, K_2x, K_3x) where K_1, K_2, K_3 are independent of x, y, z. If fluid is nonNewtonian, the stress components, in absence of hydrostatic pressure, are given by formulas of the types

$$t_{xx} = 2\Theta K_1 + 2\Psi K_1^2, \quad t_{yz} = 0,$$

wherein Θ and Ψ are physical parameters, author naming the latter the normal-stress coefficient. For $\Psi = 0$, fluid is that of classical hydrodynamics and Θ is then viscosity. It is shown that a nonzero value of Ψ may be obtained in solutions of macromolecules or suspensions of rod-like particles.

Following Kramers [J. chem. Physics 14, p. 415 (1946)], an idealized "pearl necklace" model is adopted for high-polymer molecule which is considered to consist of n equal mass points, connected by equal rigid links.

It is further assumed: (1) That solvent exerts no force on links but exerts a force $-\zeta \mathbf{v}$ on each mass point where $\mathbf{v}$ is its velocity relative to the element of solvent in its immediate neighborhood, and ζ is a constant which depends on nature of solvent and solute; (2) that presence of molecule does not disturb flow configuration in its neighborhood. Author notes improbability of second assumption.

With these assumptions, if chain of mass points is perfectly flexible and if random orientations of its links in space are not appreciably affected by flow, it is found that $\Psi = 0$. This is the first approximation. The second is obtained from the further assumption that effect of flow of solvent on dissolved molecule is equivalent to a mechanical potential field of a certain type. Author then finds that $\Psi = 4\Theta^2/(5kTn)$, where k is Boltzmann's constant, T is absolute temperature, and n is number of molecules per cc. The solution is imagined so dilute that molecules do not interfere. If hypothesis of perfect flexibility is abandoned, the extreme case where links form a straight rigid chain leads to $\Psi = 6\Theta^2/(5kTn)$.

Paper concludes with some qualitative deductions concerning behavior of more concentrated solutions.

L. M. Milne-Thomson, England

2387. **T. H. Havelock, The resistance of a submerged cylinder in accelerated motion,** Quart. J. Mech. appl. Math. II, 419–427 (Dec. 1949).

Solution of the complex boundary-value problem of resistance to motion of a circular cylinder at a constant depth below free surface of a fluid is carried to a higher order of approximation than that given in an earlier paper by author [Proc. roy. Soc. A 157, p. 532 (1936)]. Resistance is found to be expressible as the sum of a wave resistance for uniform velocity and an inertial resistance. Expressions, suitable to numerical computation, are worked out to the second order of approximation for special case of a cylinder being uniformly accelerated from rest. Curves are given of the two resistance components and of effective inertial coefficient, plotted against a nondimensional velocity parameter. The curves are constructed for a particular value of ratio of the cylinder to depth of its center (namely 0.316) and for different values of acceleration. Paper is entirely theoretical.

Robert O. Reid, USA

2388. **Adalbert Oudart, Study of jets, and theoretical fluid mechanics** (in French), Publ. Sci. Tech. Ministère de l'Air, Paris, no. 234, vii + 177 pp. (1949).

Author gives an extensive exposition of the mathematical developments connected with theory of jets. This includes the known results for two-dimensional jets, developed on the basis of inviscid fluid theory (incompressible, subsonic, supersonic), and the turbulent jet. Related mathematical methods, such as the hodograph method and the method of characteristics, are considered in some detail. C. C. Lin, USA

2389. **H. A. Stuart, On a molecular theory of the viscosity of solutions of small molecules** (in German), Z. Naturforsch. A 3a, 196–204 (Apr. 1948).

A molecular theory for viscosity of solutions is developed. Most of results obtained are valid for binary dilute solutions. Viscosity is found to depend upon an "interaction viscosity" which, in turn, is a function both of density of molecular packing and of impulsive exchange between molecules of solute and solvent. Some influences of association phenomena on viscosity are considered, and mixture-viscosity formulas for arbitrary concentration are given. Significance of negative viscosity is discussed, and some of results developed are applied to solutions of filamentary molecules. Paul M. Naghdi, USA

2390. **H. Blok and J. J. van Rossum, Lifting of liquids by viscous forces,** Proc. seventh int. Congr. appl. Mech. 2, part 1, 31–43 (1948).

Liquid film adhering to a plane plate withdrawn at uniform speed from a liquid bath is considered. Two cases are treated: (1) No liquid removed from film; (2) all liquid removed from film at some definite height above bath, e.g., "viscous pump." Earlier results are reviewed and a new relation between mean film thickness and surface tension, viscosity, density, speed of withdrawal, and acceleration of gravity is determined by means of dimensional analysis. This relation agrees reasonably well with experimental results. John A. Lewis, USA

2391. **Abdul Jabbar Abdullah, Wave motion at the surface of a current which has an exponential distribution of vorticity,** Ann. New York Acad. Sci. 51, 425–441 (1949).

Author studies surface gravity waves in water having a current distribution due to action of wind at surface. Interplay between wind and surface waves is not studied, but only the modification of gravity waves resulting from an already established current in water. It is assumed that current due to wind dies out exponentially in depth. Gravity wave motion can therefore not be irrotational. Author solves his differential equations in various cases and compares the solutions with known solutions when no wind currents are present. Stability of motions is also discussed.

Courtesy of Mathematical Reviews J. J. Stoker, USA

2392. **J. J. Moreau, Dynamics of rotational flow** (in French), C. R. Acad. Sci. Paris 229, 100–102 (1949).

Paper deals with an extension of author's earlier work [same source, 226, p. 1420 (1948); REV 1, 1239] on fluid dynamics in which generalized theorems were developed for calculation of force and moment acting on a body in an unlimited incompressible fluid. Present note gives a similar treatment of the case of a limited expanse of fluid and leads to comparable results. In particular, the force (or lift) theorem becomes a generalization of classical work of Joukowski for the airfoil case.

M. J. Thompson, USA

2393. F. Riegels, The problem of incompressible potential flow about bodies, II (in German), Ingen.-Arch. 17, 94–106 (1949).

[For part I cf. Ingen.-Arch. 16, 373–376 (1948); REV 3, 1507.] It is proposed to treat flow about an arbitrary profile by distributing singularities along an axis lying within profile. This technique is commonly used in so-called thin airfoil theory. Paper represents an attempt to extend it to thicker shapes. Formulas are derived by a kind of successive-approximation procedure and their accuracy is not clear. Force and moment are finally calculated, a procedure for numerical computation is set up, and tables of coefficients are given. A number of examples are treated, including ellipse, circular arc, and symmetrical Joukowski airfoil. In some of these cases results agree in detail with exact results obtainable by conformal mapping. In others, results resemble thin-airfoil results, having, for example, singularities at leading edge.

W. R. Sears, USA

2394. B. Thwaites, A theoretical discussion of high-lift aerofoils with leading-edge porous suction, Rep. Memo. aero. Res. Counc. Lond. R. & M. 2242, 9 pp. (1949).

Investigation is made of possibility of obtaining high lift coefficients with conventional airfoils of rather large nose radius by means of suction over a porous section of airfoil surface extending from leading edge over first few per cent of chord. Study is made on basis of boundary-layer momentum equation. It is concluded that high lift is obtained more economically in above way than by using slots at or near leading edge of an airfoil with relatively small nose radius.

William Pell, USA

2395. A. F. van Everdingen and W. Hurst, Application of the Laplace transformation to flow problems in reservoirs, J. petrol. Tech. 1, 305–324 (Dec. 1949).

By assuming that flow of fluids in permeable media obeys diffusivity equation, and that density of fluid is related exponentially to pressure, then the governing equation for flow in media of constant permeability reduces to the differential equation for flow of heat in a media of constant thermal properties. Paper is concerned with solutions of this equation for plane radial flow. Two primary cases are treated: In one, called "constant terminal pressure case," pressure at inner boundary is lowered by unity at zero time and kept constant thereafter. In the other case, called "constant terminal rate case," pressure gradient at inner boundary is changed by unity at zero time and kept constant thereafter.

Solutions for these cases for various outer boundary conditions are made by means of Laplace transformations. Some of resulting integrals are evaluated by contour integration in complex plane, and others are evaluated numerically, using Simpson's rule. Extensive tables are given.

Since the differential equation is linear, solutions may be superimposed. Therefore, by means of Duhamel's integral or its approximately equivalent series expression, solutions may be obtained for either arbitrarily variable pressures or arbitrarily variable pressure gradients at inner boundary. For this purpose the given tables are valuable.

Gerald Pickett, USA

Compressible Flow, Gas Dynamics

(See also Revs. 2192, 2370, 2377, 2378, 2379, 2388, 2449, 2459, 2460, 2469, 2471, 2479, 2484, 2490, 2491, 2502)

2396. Edward R. C. Miles, Supersonic aerodynamics, New York: McGraw-Hill Book Co., Inc., 1950. xi + 225 pp. Cloth, 6 × 9 in., $4.

This textbook, addressed to graduate students in aeronautical engineering and practical aerodynamicists, treats the subject from point of view of practical mathematics, based upon a knowledge of elements of calculus and simple notions of fluid dynamics. In introductory chapters, necessary thermodynamics of ideal gases, and concepts of divergence and circulation in gas dynamics are treated. After derivation of equations for potential and stream function, Busemann's theory of plane characteristics is exposed.

This chapter is followed by others on the linearized theory of two- and three-dimensional flow. Theory of shock waves and conical flow is presented at great length. Following conical flow, method of characteristics in axially symetric flow is given. Some notions of hodograph plane theory and a comparison between adiabatic and shock flow in two dimensions conclude the book.

Although book covers only these subjects, and does not give a complete theory of supersonic flow, it is valuable as a useful introduction to numerical treatment of supersonic-flow problems. Exposition is clear and thorough and illustrated by numerical examples, which are worked out fully. Numerous problems are included, which either give more details of theory, or are exercises in topics just treated.

R. Timman, Holland

2397. Kō Tamada, Studies of the two-dimensional flow of gas, with special reference to the singularities of the solutions of a certain partial differential equation of mixed type (in Japanese), Studies in mathematical Physics, Iwanami Shoten Publ. Tokyo 1, 82–92 (1950).

Hypothetical gas introduced by author satisfies equation $\psi_{ww} - 2kw\psi_{\theta\theta} = 0$ (see following review), which, on putting $w = (2k)^{-2/3}z$, reduces to $\psi_{zz} - z\psi_{\theta\theta} = 0$. Obtaining particular solutions in forms

$$\psi = \begin{cases} z(\theta - i\lambda)^{-2(1-\gamma)/3}F[1-\gamma)/3,\ 5/6 - \gamma/3,\ 4/3;\ 4/3; \\ 4z^3/q(\theta - i\lambda)^2], (\theta - i\lambda)^{2\gamma/3}F[-\gamma/3,\ 1/2 - \gamma/3,\ 2/3; \\ 4z^3/q(\theta - i\lambda)^2], \end{cases}$$

γ, λ being arbitrary constants, author discusses singularities of ψ which are given by $z^3 - q(\theta - i\lambda)^{2/4} = 0$. For subsonic case (elliptic type), there is a branch-point $\theta = 0$, $z = -(q\lambda 2/4)^{1/3}$, while for supersonic case (hyperbolic type), there are two branch-lines $\theta = \pm(2/3)z^{3/2}$. It is noted that, for special values of γ (e.g., $\gamma = -3/2, -1/2, -3, -2, 1/2$), ψ can be expressed in terms of elementary functions. Separate treatment is then given to case of logarithmic singularity (corresponding to $\gamma = -1/4$). Finally, it is suggested that a suitable combination of particular solutions represents a uniform flow past a lenticular-shaped body with cusped edges.

Isao Imai, Japan

2398. Kō Tamada, Studies of two-dimensional flow of gas with special reference to the flow through various nozzles (in Japanese), Studies in mathematical Physics, Iwanami Shoten Publ. Tokyo 1, 56–80 (1950).

Equations for two-dimensional flow of gas can be expressed as:

$$\Theta_\phi = w_\psi,\ \Theta_\psi = -Xw_\phi,$$

with $w = {}_1\int^q(\rho/q)dq$, $X = -(q^2/\rho)d(1/\rho q)/dq$, where q, Θ are magnitude and direction of velocity, ρ density, ϕ velocity potential, and ψ stream function. Near critical state ($q \doteq 1$),

$$X = -(\gamma + 1)w + O(w^2),$$

γ being ratio of specific heats. Author considers a hypothetical gas with property: $X = -(\gamma + 1)w$, for which basic equations reduce to $w_{\psi\psi} = k(w^2)_{\phi\phi}$, where $k = (\gamma + 1)/2$. Various examples of exact solutions are given: (1) spiral flow, (2) flow of Meyer type through a Laval nozzle, $kw = a(\phi + a\psi^2)$, a being a constant, (3) flow through a nozzle with two throats,

$$kw = \Psi_0(\psi) + \Psi_1(\psi)\phi^2,$$

Ψ_0 and Ψ_1 being certain elliptic functions, (4) flow with limit lines,

$$kw = \Phi(\phi)\Psi(\psi),$$

Φ and Ψ being certain elliptic functions, (5) flow through a Laval nozzle, $kw = Z(\phi + \psi^2) + 2\psi^2$, $Z(u)$ being defined by

$$(Z - 2u)^2(Z + u) = \text{const.}$$

In each case, detailed numerical calculations are carried out and graphs of streamlines and Mach lines (as well as limit lines and branch lines if present), both in physical and in hodograph planes are given. Last example presents particular interest, as it shows clearly the continuous transition from Taylor-type (symmetrical) to Meyer-type (asymmetrical) flow in Laval nozzle with increasing flow velocity. Isao Imai, Japan

2399. **C. S. Gardner and H. F. Ludloff, Influence of acceleration on aerodynamic characteristics of thin airfoils in supersonic and transonic flight,** J. aero. Sci. 17, 47–59 (1950).

The problem attacked is that of uniform deceleration of a thin airfoil from supersonic to transonic speed, taking advantage of fact that for sufficiently high acceleration, transonic flow about slender bodies may be linearized [C. C. Lin, E. Reissner, H. S. Tsien, J. Math. Physics 27, 220–231 (1948); Rev 3, **120**]. In supersonic range the potential function due to acceleration is calculated and expanded in powers of a dimensionless parameter proportional to acceleration. It is found that acceleration has little effect. In transonic range aerodynamic characteristics are then calculated for a diamond-shaped airfoil (two wedges, back to back). These are expressed in terms of elliptic integrals which are then expanded in powers of acceleration parameter. For one value of the acceleration parameter, coefficient of lift for a flat plate and coefficient of drag for a wedge are plotted against Mach number M for several aspect ratios. As $M \to 1+$, these coefficients approach a finite maximum which decreases with decreasing aspect ratio.

Courtesy Mathematical Reviews E. Pinney, England

2400. **John W. Miles, On the reduction of unsteady supersonic flow problems to steady flow problems,** J. aero. Sci. 17, p. 64 (1950).

Magnaradze and Galin [Galin, Prikl. Mat. Mekh. 11, 383–386 (1947); Rev 1, **532**; for a translation of these see Rev 3, **121**] have shown that if $\psi(x, y, z)$ is a solution to the wave equation $\Box^2\phi = \phi_{xy} + \phi_{zz} - \phi_{xx} = 0$, a solution of $\Box^2\phi = K^2\phi$ is given by

$$\phi(x, y, z) = \psi(x, y, z) - {}_0\int^x \psi(\xi, y, z)\frac{\partial}{\partial\xi} J_0\left[k\,(x^2 - \xi^2)^{1/2}\right] d\xi$$

It is shown how this can be used to construct solutions for oscillating supersonic wings with supersonic trailing edges from equivalent steady-flow problems. A similar method has been proposed by Germain and Bader [C. R. Acad. Sci. Paris 228, 1201–1202 (1949); Rev 2, **1160**]. W. R. Sears, USA

2401. **G. Gontier, Instrument for plotting of plane supersonic flows** (in French), Recherche Aéronautique 1949, no. 12, 3–9 (1949).

Author has devised a drawing instrument which he calls a "caragraph" to simplify and render routine the graphical solution of problems in two-dimensional steady compressible flow of a perfect gas of adiabatic constant 1.4. Design of caragraph depends upon the fact that Mach lines in plane of flow are orthogonal to characteristics in hodograph plane, and that these characteristics are congruent epicycloids. Given direction and magnitude of flow at a given point P, the corresponding point P' in hodograph plane is determined. By means of caragraph, characteristics through P' may be drawn rapidly and their normal directions determined, thus giving directions of Mach lines through P. This device is particularly adapted to problems in which field of flow is divided by Mach lines into a finite number of regions in which flow is uniform, or to problems in which field of flow may be approximated in this way. Examples of both types are given. Accuracy appears to be rather good for so simple a device.

Courtesy Mathematical Reviews E. Pinney, England

2402. **Paul Germain, General theory of conical motions and its applications to supersonic aerodynamics** (in French), Off. nat. Étud. Rech. Aéro. Rep. 34, 197 pp. (1949).

In study of small-perturbation flows at supersonic speeds there appear cases of conical flow, i.e., velocity components have the form $F(x_2/x_1, x_3/x_1)$ or $f(\theta, \chi)$, where x_1, x_2, x_3 are Cartesian coordinates; x_1 in the stream direction, $x_2 = r\cos\theta$, $x_3 = r\sin\theta$ $\chi = x_1/\beta r$; $\beta^2 = M^2 - 1$, and M is the stream Mach number. Author obtains the differential equation for flow of this type in the forms:

$$\text{for } \chi > 1\text{: } f_{\xi\xi} + f_{\theta\theta} = 0, \text{ where } \chi = \cosh\xi$$
$$\text{for } \chi < 1\text{: } f_{\eta\eta} - f_{\theta\theta} = 0, \text{ where } \chi = \cos\eta$$

He proposes to study two main types of flow: flow past slender cones, and flow past nearly flat cones (conical wings). He also points out the more general cases called "homogeneous" flows, i.e., those for which the velocity potential is $\varphi = r^n f(\theta, \chi)$, n being a nonnegative integer.

Cases involving slender cones are treated by function-theoretical methods; application is made to circular and elliptic cones, a cone whose section has a sharp corner, a slender delta wing, and a cone of semicircular section. General formulas for lift and drag are found. Problem of numerical calculation is treated in considerable detail. Passing on to conical wings, author treats separately the cases of supersonic and subsonic leading edges. Again, mathematical techniques are those of complex-variable theory. Electrical-analog methods are explained and have been used to obtain some numerical results. In final chapter, it is shown how conical flows can be used to build up flow patterns over other configurations. These include rectangular and swept-back wings, ailerons, wings with dihedral, and crossed wings.

Main part of report was prepared in 1948 and makes reference to much of pertinent literature available at that time. There are six appendixes, added to report in 1949, giving further development and applications, and referring to some more recent publications. W. R. Sears, USA

2403. **Arthur L. Jones, Robert M. Sorenson, and Elizabeth E. Lindler, The effects of sideslip, aspect ratio, and Mach number on the lift and pitching moment of triangular, trapezoidal, and related plan forms in supersonic flow,** Nat. adv. Comm. Aero. tech. Note 1916, 69 pp. (Aug. 1949).

Note is a continuation of a theoretical study of aerodynamic characteristics of finite-span airfoils at supersonic speeds as affected by sideslip. Lift and pitching moment variations with Mach number are derived on the basis of linearized potential

theory for supersonic flow for several different plan forms. Geometric plan-form parameters considered include aspect ratio, sweepback, taper and tip rake. Results show a slight but steady increase in lift and pitching moment with increasing sideslip angle for most of considered cases. Shown data are entirely theoretical, no comparison with experimental results being presented. M. J. Thompson, USA

2404. Harold Mirels and Rudolph C. Haefeli, Line-vortex theory for calculation of supersonic downwash, Nat. adv. Comm. Aero. tech. Note 1925, 54 pp. (Aug. 1949).

Field of flow round a flat airfoil at incidence under incompressible conditions may be regarded as due to a system of vortexes distributed over airfoil and its wake. Linearized theory of compressible flow leads to a similar result for supersonic conditions. If pressure distribution on airfoil is known, it is possible to determine appropriate vortex system and hence to calculate downwash aft of airfoil. In present paper various types of supersonic vortex systems are investigated in detail. It is shown in particular that—unlike in subsonic theory—discontinuities of shed vorticity may occur along span of an airfoil under supersonic conditions leading to (theoretical) infinities in downwash field aft of airfoil. As an additional approximation, authors replace surface distribution of bound vorticity by a single lifting line, which may be bent for sweptback wings, in particular delta wings. Good agreement with exact linearized theory is obtained at points which are situated more than half chord aft of trailing edge.

Authors rightly stress need for an extensive experimental program to check their results, a need which exists here as in other branches of supersonic airfoil theory. A. Robinson, England

2405. M. J. Lighthill, The shock strength in supersonic conical fields, Phil. Mag. (7), 40, 1202–1223 (1949).

One of the methods of approximate solution of the differential equations of isentropic irrotational flow is to seek the velocity potential or velocity components as power series in a small parameter ϵ defined by thickness of the body or its incidence. For example, if flow is due to a stream of speed U, the velocity potential ϕ may be written

$$\phi = U_x[1 + f(r, \theta)] = U_x(1 + f_1 + f_2 + \ldots)$$

where subscripts indicate power of ϵ involved. When the flow field is conical, the successive equations for f_i are of the form

$$(1 - r^2)(f_i)_{rr} + r^{-1}(f_i)_r + r^{-2}(f_i)_{\theta\theta} = F \qquad [1]$$

Here r denotes $(M^2 - 1)(x^2 + y^2)^{1/2}/z$, and θ denotes $\tan^{-1}(y/x)$; x, y, z are Cartesian coordinates, the z axis being the conical axis (direction of stream flow); and F involves f_{i-1}, f_{i-2}, $\ldots f_1$, and derivatives thereof. Equation [1] has a singularity at the Mach cone from origin, $r = 1$; $(f_1)_{rr}$ is infinite there, and consequently $(f_2)_r$ and all successive $(f_i)_r$ are infinite there. Thus, expansion diverges in this region and it is impossible to study the flow there, or to satisfy correct upstream boundary conditions, beyond first order.

Author presents here a method by which the singularity is avoided in each step of the successive-approximation process. He assumes a coordinate transformation

$$R = r - r_1(\theta) - r_2(\theta) - r_3(\theta) - \ldots$$

where again subscripts indicate powers of ϵ. He is then able to choose each $r_i(\theta)$ in succession so as to make the coefficient of $(f_{i-1})_{rr}$ in F vanish. This permits convergence of expansion near $R = 1$. This process is to be carried out twice; once for $R < 1$ and once for $R > 1$. But R (for given r, θ) has a different meaning in these two regions, and the regions actually overlap, so that solutions can be joined.

By this technique, author is able to satisfy the shock-wave equations in overlapping region. For cases where there is no disturbance ahead of the nose Mach cone it is found that $r_1(\theta)$ is zero. Shock strength is $O(\epsilon^2)$ at most. There are simple formulas for shock strength and position, and for $r_2(\theta)$ and other quantities, in terms of the first-order solution. Cases where there is disturbance outside the Mach cone are also treated. As detailed examples, flows past a slender cone, a flat triangular wing, and a flat rectangular wing tip are considered.

Paper represents an application of a general mathematical method developed by author [title source (7), 40, 1179–1201 (1949); REV 3, 1829]. W. R. Sears, USA

2406. W. E. Moeckel, Approximate method for predicting form and location of detached shock waves ahead of plane or axially symmetric bodies, Nat. adv. Comm. Aero. tech. Note 1921, 32 pp. (1949).

Flow about a blunt obstacle in a supersonic stream is considered. Using an approximation due to A. Busemann [same source, no. 1858; REV 3, 2001] to find intersection of sonic line and obstacle profile, assuming sonic line to be straight and shock locus to be a hyperbola, author is able to locate sonic line and shock locus, and hence to compute drag exerted by subsonic portion of stream. A comparison with experimental evidence is given. Approach differs considerably from that of Lin [REV 1, 1391], and a comparison of the overlapping portion of their results would be interesting. George Carrier, USA

2407. Henri Cabannes, Approximate determination of detached shock wave (in French), C. R. Acad. Sci. Paris 229, 492–493 (Aug. 1949).

Author starts from L. Crocco's equations [ZAMM 17, 1–17 (1937)] for stationary axially symmetric flow with constant enthalpy, without rotational velocity component but with vorticity (vortex lines perpendicular to meridian planes). Undisturbed flow has uniform supersonic velocity; this flow is diverted by an obstacle, meridian curve of which is written $x = h + r^2/2R^*$. In front of obstacle a curved shock wave is supposed to be formed, with meridian curve $x = r^2/2R$. A development is assumed for stream function of flow in region between shock wave and obstacle: $\psi = r^2\, a(x) + r^4\, b(x)$. Substitution of velocity components and other derivatives of ψ into conditions at shock wave and into Crocco's equation for ψ makes it possible to obtain coefficients of developments of a and b in terms of x. Restriction to first three terms of $a(x)$ makes it possible to calculate both h and R^* as functions of R and of Mach number of undisturbed flow.
J. M. Burgers, Holland

2408. Hermann Weyl, Shock waves in arbitrary fluids, Comm. pure appl. Math. 2, no. 213, 103–122 (1949).

This illuminating paper is in two parts. In the first, conditions are laid down which must be satisfied by the equation of state of a fluid in order that the shock phenomenon, with its usual characteristics, may occur. In the second, a gas satisfying these conditions is postulated and nature of the interior of shock layer itself is examined.

Conditions which author finds sufficient for shock formation in part I are in part local, in part topological in character. The local ones assert that pressure compresses gas, and that compression becomes more difficult as volume decreases. Topological conditions are to the effect that manifold of possible states Z must be homeomorphic to a convex region in (p, τ) plane (p pressure, τ specific volume). Then, corresponding to any state

$Z_0(p_0, \tau_0)$ there corresponds a curve $Z(s)$ with the property that each ray from Z_0 cuts it exactly once, and points $Z(s)$ comprise totality of points Z connected with Z_0 by Hugoniot relations. Thus Z_0 and $Z(s)$ are states before and after a shock, s representing some such parameter as shock strength. Usual shock properties are then established.

In part II the full equations of mass, momentum, and energy are considered with heat conduction and viscous stresses included. Two methods of allowing the thermal and stress parameters to vanish are discussed, one leading to a shock, the other to a slip-stream. Only the former is considered further. This amounts to multiplying the coordinate x and the parameters by ϵ, and then letting $\epsilon \to 0$. Resulting equations boil down to two nonlinear first-order equations in p and τ, involving the arbitrary equation of state of the gas (assumed only to obey the earlier conditions). The states Z_0 at $x = -\infty$ and Z_1 at $x = +\infty$ correspond, in view of indefinite expansion of x scale, to conditions on opposite sides of shock. Author shows that these are both singular points and indeed the only singular points of the system of differential equations. Moreover he proves that initial point Z_0 is an unstable node, final point Z_1 a saddle point. Author points out that this particular combination of types would lead one to expect unicity of solution. In fact, L. A. MacColl has indicated to reviewer that uniqueness can be proved without further assumption. Moreover, existence is assured if one is willing to discard certain topological possibilities which correspond to highly improbable physical situations. D. P. Ling, USA

2409. **Luigi Crocco, A new potential function for the analysis of two dimensional non-isentropic gas motion** (in Italian), Aerotecnica 29, no. 6, 347–355 (Dec. 1949).

Paper deals with the general problem of steady two-dimensional adiabatic flow of a gas in which, owing to presence of shocks, entropy is nonuniform. Author points out, following Reynolds, that the quantities $p + \rho u^2$, ρuv, $p + \rho v^2$, where p is pressure, ρ density, and (u, v) velocity, satisfy equations of equilibrium of a distribution of plane stress. He then introduces $-\phi$ as the Airy stress function of this stress distribution (so that the three quantities mentioned are $-\phi_{yy}$, ϕ_{xy}, $-\phi_{xx}$), and calls ϕ a new "potential" of this (nonirrotational) flow. From its definition, and from the Bernoulli equation, quantities p, ρ, u, and v are all expressible algebraically in terms of ϕ_{xx}, ϕ_{xy}, ϕ_{yy}. Equation of continuity then gives a quasilinear third-order partial differential equation for ϕ, whose three systems of characteristics are, of course, the Mach lines and streamlines.

As this equation is somewhat complicated, an alternative formulation of equations of motion is given, using stream function ψ and new potential; in this formulation the equations consist simply of $\psi_x = \rho v$, $\psi_y = -\rho u$, with the right-hand sides replaced in terms of ϕ. Shock conditions may be expressed very simply, by saying that ϕ, ϕ_x and ϕ_y are continuous across a shock. Same is true for a vortex sheet. Outside a finite solid cylinder ϕ, ϕ_x and ϕ_y may return to different values on a path encircling it once; changes involved are simply related to forces and moment on unit length of cylinder. Paper ends with treatment of various known gas flows by new method.

M. J. Lighthill, England

2410. **J. Kestin and J. S. Glass, Application of the method of characteristics to the transient flow of gases,** Proc. Instn. mech. Engrs. 161, 250–258 (1949).

Equation of characteristics in one-dimensional unsteady isentropic flow of an ideal gas are established in a quite elementary way by studying propagation of weak disturbances. Author then applies method of characteristics (in form due to P. Haller) to shockless flow patterns resulting from motion of a piston in a pipe of infinite length and in a pipe of finite length open to atmosphere. Same subject has been treated formerly by reviewer [Z. Ver. deutsch. Ing. 88, p. 301 (1944)]. R. Sauer, USA

2411. **C. C. Lin, Note on the characteristics in unsteady one-dimensional flows with heat addition,** Quart. appl. Math. 7, 443–445 (Jan. 1950).

Method of characteristics with heat addition is formulated in its simplest and most convenient way for numerical computation.

H. S. Tsien, USA

2412. **N. Coburn, Degenerate two-dimensional non-steady irrotational flows of a compressible gas,** Quart. appl. Math. 7, 439–443 (Jan. 1950).

Author considers degenerate group of two-dimensional non-steady irrotational flows of a compressible gas for which characteristic surfaces are cylinders with generators parallel to the time axis. Aside from steady flows, which are obviously members of this group, author finds two results. First, simple waves do not exist for nonsteady degenerate flows. Second, one of the non-steady degenerate flows has streamlines which are logarithmic spirals and a uniform Mach number, $M = 5^{1/2}$, throughout entire flow region. H. J. Stewart, USA

2413. **Mac C. Adams, On shock waves in inhomogeneous flow,** J. aero Sci. 16, 685–690 (Nov. 1949).

Paper treats effect of small flow disturbance on a standing normal shock wave in a channel flow. Local distortion of shock wave generates vorticity in subsonic flow of shock downstream. First-order magnitude of such vorticity is considered with differential equation derived by Sears.

A few interesting points may be mentioned. Ratio of horizontal velocity components, positive disturbance ahead of shock and negative disturbance behind shock are same as that of flow velocities ahead and behind shock. Vorticity in subsonic flow behind shock is, in first order, proportional to velocity gradient of disturbance ahead the shock. For sinusoidal velocity disturbance, distorted shock configuration is shown. Effect of vorticity becomes more pronounced at higher Mach number. For roof-top disturbance, distorted configurations of shock are shown at different Mach numbers. At a particular Mach number $M = (\gamma + 3/2)^{1/2}$, flow is found to remain parallel before and after shock for continuous velocity profile of disturbance.

Chieh-Chien Chang, USA

2414. **Carlo Ferrari, Interference between wing and body at supersonic speeds—analysis by the method of characteristics,** J. aero. Sci. 16, 411–434 (1949).

Flow field in neighborhood of body-wing combination at zero-yaw, but positive incidence, is systematically investigated in a step-by-step method based on existence of characteristic surfaces. This presupposes supersonic flow everywhere; it further assumes that head shock wave is attached to tip of body and that ensuing flow may be considered as irrotational, but no linearizations are introduced. Paper thus supplements a previous paper [J. aero. Sci. 15 (1948); Rev 1, **1268**], in which author treated the linearized problem.

Present work consists of four major parts, viz., the determination of flow fields (a) along conical nose, (b) along fuselage as far as it is free from wing interference, (c) around wing as far as it is affected by fuselage ahead (wing tips, however, are assumed outside interference region), and (d) along that part of fuselage which is affected by wing.

Analytical descriptions of flow, although in principle the same

in parts (b), (c), (d), differ in coordinate systems used. In part (a) a quasiconical field [$\phi = rF(\varphi, \vartheta)$, φ = cone angle, ϑ = azimuth] is assumed. No cylindrical symmetry, but a vertical symmetry plane $\vartheta = \pm \pi/2$ is present; dependence on ϑ, however, is weak. Hodograph of velocity components in a meridian plane ϑ = const is introduced, and (approximate) validity of Busemann's formula for hodograph curvature in conical flow is assumed. It then becomes possible to determine $\mathbf{v}$ for any meridian plane, starting from the intersection of the conical surface $\varphi = f(\vartheta)$ with the meridian plane, provided $\mathbf{v}$ is given all along that surface. It is possible either to identify $f(\vartheta)$ with the head shock wave and to proceed inward to new conical surfaces until a possible nose boundary ($v_n = 0$) is reached, or to go outward from a given nose until the shock conditions are fulfilled.

Field determination in parts (b), (c), (d) is carried out directly by method of characteristics. In (b) cylindrical coordinates are used, and step-by-step determination is carried out in meridian planes, "characteristic lines" in these planes being defined as intersections of the two systems of characteristic surfaces Σ and Σ'. Body contour is replaced by a sequence of straight segments. It is shown that flow in proximity of corners may be considered a simple wave in each meridian plane. Field in portion (c) is analyzed in coordinates x, Y, Z, the characteristic lines being intersections of Σ and Σ' with planes Y = const and Z = const. (Y axis coincides with wing span.) On upper wing surface an expansion field is determined in each of the set of planes Y = const. On the lower surface one starts with a shock wave of shape $x = Zg(Y)$, which must be brought in accordance with the upstream field (b). Boundary condition at the body is circumvented by the expedient of assuming the planes $Y = \pm R$ (R is the fuselage radius) coincident. The field in (d) is again analyzed in cylindrical coordinates, and characteristic lines in planes ϑ = const and cylinders y = const utilized. Several simplifications are suggested for this part to make numerical work less cumbersome.

Paper is self-contained in that it gives the derivation of almost entire mathematical apparatus required. In particular, parts (a) and (b) point out all steps in detail; part (b) brings a numerical example. Methods employed are essentially generalizations of Sauer's method for axisymmetric flow. G. Kuerti, USA

2415. Ralph E. Grey, Jr. and H. Dean Wilsted, Performance of conical jet nozzles in terms of flow and velocity coefficients, Nat. adv. Comm. Aero. Rep. 933, 9 pp. (1949).

See Rev 2, 211.

2416. Roger Weatherston, Mixing of any number of streams in a duct of constant cross-sectional area, J. aero. Sci. 16, pp. 697–698, 704 (Nov. 1949).

Mixing of any number of streams of a compressible nonviscous fluid in a duct of constant cross-sectional area is treated by means of a one-dimensional theory. Solution is given in terms of four quantities: temperature, mass flow, Mach number, and cross-sectional area. Final result is a simple equation which gives N_f (a function of the Mach number after mixing) in terms of initial conditions in the various streams. A plot of Mach number against N_f is given from which the Mach numbers after mixing can be read off. Bruno A. Boley, USA

2417. J. A. Zaat and R. Timman, Calculation methods for profiles in subsonic compressible flow (in Dutch), Nat. Luchtlab. Amsterdam Rap. F. 57, 37 pp. (Aug. 1949).

Lin's method for representation of a compressible subsonic flow of a fluid with a linear adiabatic relation ($p = A - B/\rho$) is applied to two principal problems of airfoil theory, viz., design of airfoils with a velocity distribution specified at a given angle of incidence and a given Mach number, and computation of compressible flow about a given airfoil contour; both were solved for incompressible flow in previous reports. The straight line, which approximates actual adiabate for air, is taken through point corresponding to critical velocity of sound and point corresponding to undisturbed flow. It was expected that in this way a good approximation in region of high subsonic velocity could be obtained. Results are compared with calculation by relaxation methods of Howard (NACA T.N. 1746) for NACA 0012 airfoil. No definite improvement over the known von Kármán-Tsien approximation is obtained. R. Timman, Holland

2418. A. K. Oppenheim, Dynamics of combustion or heat addition to a compressible flow, Proc. seventh int. Congr. appl. Mech. 3, 305 (1948).

Dynamic stability of combustion waves, restricted only to one-dimensional treatment, is analyzed by a special formulation of combustion process (dynamic analogy) which leads to representation of changes of state associated with combustion waves by a reversible heat addition at constant impulse per unit area (constant "specific thrust"). Process is represented by a Rayleigh line—a straight line on P-v diagram.

In development of combustion from a "non-flow burning" (corresponding to a diffusion flame) to a fully developed detonation wave, following regimes are distinguished: (1) Weak deflagration, (2) fully developed deflagration, (3) weak detonation, and (4) fully developed detonation.

By consequent application of dynamic principles it is proved that appearance of detonation wave is associated with generation of shock wave in front of it. While thermodynamic states immediately behind the waves of regimes 1, 2, and 4 lay on Hugoniot curve, states behind waves of regime 3 form a new locus of points determined by condition that velocity of propagation of combustion wave with respect to burnt gases is equal to local velocity of sound. By use of this representation, proof of dynamic stability of fully developed detonation wave is developed. From author's summary

2419. George R. Costello, Method of designing airfoils with prescribed velocity distribution in compressible potential flows, Nat. adv. Comm. Aero. tech. Note 1913, 25 pp. (Aug. 1949).

Under assumption of linearized equation of state, author solves inverse problem to that of obtaining velocity distribution of a two-dimensional potential flow of a perfect fluid about a preassigned airfoil shape; namely, given a preassigned theoretically attainable velocity distribution, to design an airfoil shape which will give rise to this distribution. If velocity distribution gives rise to a nonclosed curve as a body, author's method enables one to modify distribution to satisfy closure condition for body.

Method is essentially a means of fixing the arbitrary analytic function in the formula

$$z = f(\zeta) + \int^{\zeta} G'^2(\zeta)/f'(\zeta)\, d\zeta,$$

where $G(\zeta)$ is fixed and $f(\zeta)$ belongs to a wide class of analytic functions, so that mapping from ζ-plane to z-plane will yield the desired body; this formula was first established by the reviewer [NACA T.N. 1170 (1947)] and independently by Lin [Quart. appl. Math. 4, 291–297 (1946)]. A. Gelbart, USA

2420. C. R. Illingworth, Steady flow in the laminar boundary layer of a gas, Proc. roy. Soc. Lond. Ser. A 199, 533–558 (Dec. 1949).

Author gives a rather complete discussion of application of the

von Mises transformation to equations of steady, compressible, laminar boundary layer with pressure gradient about a two-dimensional cylinder. Although potential flow is assumed normal to leading edge of cylinder, gravity body forces are considered and are shown to result in a spanwise secondary flow in the boundary layer when these forces have a spanwise component. Effect of these secondary flows on the primary chordwise flow is considered small.

After the general equations in terms of the von Mises independent variables (stream function and distance along surface of cylinder) are established, several primary flows are considered. An extension of the von Kármán-Tsien method of successive approximations for flat plate flow with general but constant Prandtl number, and for a general viscosity-temperature relation, is outlined. This method consists of converting the governing ordinary differential equations into integral equations, which are to be solved by successive approximation starting from the solution with a linear viscosity-temperature relation.

Flow near the leading edge of a blunt-nosed cylinder is considered next. By neglecting viscous dissipation, a method of solution for general Prandtl number and general viscosity-temperature relation is presented, again involving solution of an integral equation by successive approximations, starting in this case from the incompressible flow. Neglect of viscous dissipation is justified in the neighborhood of a stagnation point where the potential flow velocity is small.

For the primary flow in the boundary layer of a heat-insulated cylinder, author shows quite readily that this problem for compressible flow reduces to a related problem of incompressible flow, provided the Prandtl number is unity and viscosity-temperature relation linear. Same result was found independently by Stewartson [Proc. roy. Soc. 200, no. 1060 (Dec. 1949)] by a different set of transformations. Reviewer points out, however, that solution of the incompressible case with general pressure gradient represented by a high-order polynomial is cumbersome, and that the essential simplification achieved by reduction to incompressible flow is questionable. For the heat-transfer case, a set of second-order linear differential equations with variable coefficients must be solved, after the same equations as for the flat plate case are solved.

Finally, several flows with body forces considered are discussed. Secondary flows in boundary layers of a vertical flat plate and of a vertical blunt-nosed cylinder, both in horizontal flow, are shown to be small compared to the primary flow. This result checks *a posteriori* assumption that secondary flow has a negligible effect on primary flow. Paper concludes with a brief discussion of use of the von Mises transformation in analysis of boundary layers in plane jets and wakes.

Although a minimum of actual calculations is given, paper will be of interest to all concerned with laminar boundary-layer theory. Paul A. Libby, USA

2421. A. M. Binnie, The passage of a perfect fluid through a critical cross-section or "throat," Proc. roy. Soc. Lond. Ser. A 197, 545–555 (July 7, 1949).

Author calculates velocity in critical cross section of flow of a perfect fluid under various conditions of flow. Critical cross section is understood to be the one in which velocity of current is equal to velocity of sound for gases, and to velocity of a long wave for liquids. For determination of such velocity, Hugoniot method is applied. It is demonstrated that if external forces do not have components in flow direction, velocity of sound is attained by fluid at exact geometrical throat of constriction. In the contrary case, condition which determines position of critical cross section is indicated. Carlo Ferrari, Italy

2422. A. D. Young, Skin friction in the laminar boundary layer in compressible flow, Aero. Quart. 1, 137–154 (Aug. 1949).

Laminar boundary layer associated with flow of gas over a semi-infinite plate has been studied by Crocco, Hantzsche and Wendt, and others. It is known that skin friction coefficient c_f can be expressed by a formula of the type

$$c_f/(R_x)^{1/2} = f(\gamma, M_1, i(0), \sigma, \omega) \qquad [1]$$

where R_x is Reynolds number based on distance from leading edge of plate, γ ratio of specific heats, M_1 free-stream Mach number, $i(0)$ ratio of enthalpy (or temperature) at wall to that of free stream, σ Prandtl number, and ω exponent in variation of viscosity with temperature. Author develops the semi-empirical formula

$$c_f/(R_x)^{1/2} = 0.664\,[0.45 + 0.55\,i(0) + 0.09\,(\gamma - 1)\,M_1^2\sigma^{1/2}]^{1/2(\omega-1)} \qquad [2]$$

by making an approximation to Crocco's integral equation for viscous stress and using experience deduced from elaborate numerical calculations of previous authors. For example, in case of no heat transfer it has been found by Crocco and others that $c_f/(R_x)^{1/2} = G(0)/[1 + (\gamma - 1)M_1^2\,\sigma^{1/2}/2]^{(1-\omega)/2}$ is a good approximation for a range of values of ω. Also, according to numerical results of Hantzsche and Wendt, $G(0)$ varies but slightly with σ, ω, and M_1. Constants in final formula [2] have been adjusted to fit calculated results to give an accuracy of 1% up to a Mach number of 10.

Problem of heat transfer and effect of radiation are discussed with help of these formulas. Momentum equation of boundary layer on a cylinder with no heat transfer is solved by making certain assumptions. Method is correct for case of no pressure gradient and is quite simple in presence of a pressure gradient. However, author states that "accuracy of the method cannot yet be gaged." In applying method to calculate point of separation, it agrees qualitatively, but not quantitatively, with result of Howarth in predicting a rapid forward movement of separation with increase of Mach number. C. C. Lin, USA

2423. W. W. Fischer and R. H. Norris, Supersonic convective heat-transfer correlation from skin-temperature measurements on a V-2 rocket in flight, Trans. Amer. Soc. mech. Engrs. 71, 457–469 (July 1949).

Coefficient of convective heat transfer has been evaluated by analysis of skin-temperature measurements made at various points on nose of a V-2 rocket during flight. Comparison is made between test results in dimensionless form, and various proposed correlations. Comparison indicates that transition from laminar to turbulent conditions depends more, at supersonic speeds, on Mach number than on Reynolds number. Test results confirm the theoretical values for laminar layer. Subsonic correlation for flat plates gives better agreement with supersonic test results than that proposed by G. Eber [*Experimental research on friction, temperature, and heat transfer for simple bodies at supersonic velocities*, Köchel Archiv 66/57 (Nov. 21, 1941)].
R. M. Wingren, USA

2424. Morris Morduchow and Paul A. Libby, On a complete solution of the one-dimensional flow equations of a viscous, heat-conducting, compressible gas, J. aero Sci. 16, 674–684, 704 (Nov. 1949).

Hydrodynamic equations describing steady, one-dimensional flow of a viscous, heat-conducting, compressible gas are treated. The equation of state for a perfect gas is assumed valid, and coefficients of specific heat are assumed constant. A complete integral of energy equation is found for a Prandtl number of 3/4,

which is a good approximation for actual value of many gases over wide temperature ranges. A particular integral contained in this complete integral is $(u^2/2) + c_pT = \text{const}$, which is a frequently used one-dimensional energy equation. Restrictions on its use are rigorously derived and stated. With the complete integral of energy equation, three different types of solutions which depend on boundary conditions are shown to be obtainable. The usual shock-wave solution (satisfying the Rankine-Hugoniot relations) is only one of these three types.

Several features of shock-wave solution are shown, e.g., entropy has a maximum value at the inflection point in velocity distribution; validity of continuum hypothesis is examined, and it is shown that thickness of wave as defined by Prandtl becomes infinite as the initial Mach number of flow becomes infinite, provided that n, the exponent in temperature-viscosity relation $\mu/\mu_0 = (T/T_0)^n$, is greater than 1/2. Furthermore, it is concluded that the continuum theory may give reasonably correct results for flows with Mach numbers up to 1.3.

Other two cases of flows contained in the equations are terminating flows, i.e., flows that can be valid only in a finite region. Their mathematical nature is demonstrated by two numerical examples, which indicate compression shocks followed by expansion waves. However, the physical significance, if any exists, of such solutions remains open to question.

J. Howard Childs, USA

2425. H. A. Johnson and M. W. Rubesin, Aerodynamic heating and convective heat transfer: summary of literature survey, Trans. Amer. Soc. mech. Engrs. 71, 447–456 (July 1949).

Literature represented by 37 references is surveyed. It is stated that high-speed convective heat transfer can be calculated by the equation: $q = h[t_w - (t_0 + r\,u_0^2/2c_p)]$, where q is rate of heat transfer per unit area, h heat-transfer coefficient, t_w surface temperature, t_0 stream temperature, "recovery factor" $r = (t_{aw} - t_0)/(u_0^2/2c_p)$, u_0 free-stream velocity, c_p specific heat at constant pressure, and t_{aw} temperature reached by an insulated surface in a stream of velocity u_0.

Theoretical and experimental results for recovery factors for laminar and turbulent subsonic and supersonic flows over flat plates, wedges, and cylinders are presented and discussed. For laminar flow, it is concluded that relation $r = P_r^{1/2}$, where P_r is Prandtl number, is a good approximation for most body shapes. Recovery factor for turbulent flow appears to depend on Mach number, in contrast to that for laminar flow, and to exceed a suggested value of $P_r^{1/3}$.

Theoretically derived equations and a few measurements are given for heat-transfer coefficients of laminar and turbulent flow. It is concluded that heat-transfer coefficient, h, is independent both of modified temperature potential $[t_w - (t_0 + r\,u_0^2/2c_p)]$ and of frictional-dissipation effects. [Reviewer's note: It is believed that conclusion that h is independent of compressibility effects has not been substantiated. It can be shown, at least for flat plates, that h varies directly with skin-friction coefficient, and therefore depends on Mach number and on ratio of plate to stream temperature whenever skin friction does.]

Neal Tetervin, USA

2426. W. F. Cope, Heat transfer at high speeds, Proc. seventh int. Congr. appl. Mech. 3, 120–126 (1948).

It is shown that for laminar boundary-layer flow along a flat plate, the law of heat transfer is same for high speeds as for low, if wall temperature is substituted for temperature of main stream when heat transfer is zero. It is argued that same result should apply to cases of flow with pressure gradient and a turbulent boundary-layer flow.

Author refers often to a more extensive treatment of the problem by Cope and Hartree [Phil. Trans. roy. Soc. A 241, 1–69 (1948); REV 2, 1169]. G. M. Lilley, England

2427. A. H. Flax, Relations between the characteristics of a wing and its reverse in supersonic flow, J. aero. Sci. 16, 496–504 (Aug. 1949).

Author shows that in linearized supersonic flow a wing has same lift curve slope, damping in roll, damping in pitch, and thickness drag as the same wing with the flow reversed, with severe restriction of no subsonic edges. More general theorems for drag are available [Hayes, North American Aviation Rept. AL-222 (1947)] and, recently, C. Brown has given a much more general proof of theorems for lift and related quantities.

Wallace D. Hayes, USA

2428. John Stack and W. F. Lindsey, Characteristics of low-aspect-ratio wings at supercritical Mach numbers, Nat. adv. Comm. Aero. Rep. 922, 6 pp. (1949).

Authors point out that determination of wings suitable for flight at high subsonic speeds is based largely on attempt to avoid rapid drag rise, loss in lift, and resultant stability changes which are brought about by flow fields associated with supercritical Mach numbers. Results are given of tests made in the Langley 24-in. high-speed tunnel of rectangular plan forms with aspect ratios ranging from ∞ to 2. Wings had NACA 0012 sections; Mach number range was from 0.50 to 0.9; Reynolds number varied from 5.3×10^5 to 7.6×10^5. The experiments indicate that favorable relieving effects due to three-dimensional flow around tips are of such magnitude that low-aspect-ratio wings offer substantial aerodynamic gains. Max A. Heaslet, USA

2429. C. A. Truesdell, On Ertel's theorem of the diffusion of vorticity, Nav. Ord. Lab. Mem. 9528, 8 pp. (1948).

A general diffusion equation involving gradient of an arbitrary function is deduced. It is then specialized to case of a viscous compressible fluid. When viscosity may be neglected, it reduces to a theorem of Ertel [Naturwissen. 30, 543–544 (1942)]. From Ertel theorem it is shown that mixing of two inviscid incompressible fluids leads to a motion which is unstable, except in special cases of plane and axially symmetric motion. Result may explain why surface of separation for a spherical explosion in water is much smoother than that for a similar explosion in air. Optimum form for estuary where a fresh-water stream meets the ocean is discussed. Ertel theorem is applied also to show that specific vorticity generated at head shock of a slightly yawed projectile increases as air passes into wake.

From author's summary by C. C. Lin, USA

2430. D. W. van Krevelen and J. Huiskamp, The technology of mixing (in Dutch), Ingenieur 's-Gravenhage 61: no. 31, 6–12; no. 34, 15–22 (Aug. 1949).

Paper is a contribution to problem of giving a more fundamental basis for design and use of mixers. Authors compare results of some 20 experimenters on power consumption and effectiveness. Distinction is made between mixing with axial, radial, and tangential flow. For each of these types a graph is given, relating dimensionless "power-number" P/n^3L^5 to a kind of Reynolds number $nL^2\,\rho/\eta$, where P is power consumed, n rotational speed of stirrer, L its length, ρ density, and η viscosity of liquid. All curves have same form, showing a laminar and a turbulent region. Some instances are mentioned of influence of geometrical form of stirrer and arrangement on power consumption and on intensity and distribution of circulation.

Further, rates of dissolving, of melting, and of heat transfer in

agitated liquids, as found by different authors, are given the form of Kd/D or $\alpha d/\lambda$ as a function of the Prandtl number $\eta c_p/\lambda$, the Schmidt number $\eta/\rho D$, and the Reynolds number $nL^2\rho/\eta$, where d is diam. In the case of Hixson and Baum's measurements, the values of K and α were calculated from the measured concentrations and temperatures, with the aid of approximate solutions of differential equations for processes of dissolving and melting. The functions mentioned are given in the form of curves and formulas of the type $Kd/D = \text{const}\ (nd^2\rho/\eta)^m \cdot (\eta/\rho D)^n$.

B. H. Schultz, Holland

Turbulence, Boundary Layer, etc.

(See also Revs. 2374, 2385, 2422, 2423, 2426)

2431. **H. Reichardt, On the question of shear-stress measurements in a turbulent flow** (in German), Z. angew. Math. Mech. 29, 16–18 (Jan.-Feb. 1949).

It is suggested that a direct measurement of shear stress in turbulent flow can be made by observations on the mean normal force acting on a very small plate with leading edge facing mean flow direction, and at zero angle of attack. Associated experimental difficulties are described and preliminary observations reported. Newman A. Hall, USA

2432. **J. Rotta, New calculations on statistical theory of turbulence** (in German), Z. angew. Math. Mech. 29, 12–14 (Jan.-Feb. 1949).

Author attempts to extend Heisenberg's theory to include effect of large turbulent elements. He assumes that shape of spectrum of a decaying turbulence does not change during a small interval of time. Spectrum is given by $F(k) = E(t)\cdot L(t)\cdot f(\chi)$; E is energy and L a characteristic length of turbulence, each a function of time t. $f(\chi)$ is a function of $\chi = kL$ assumed to remain invariable during a small interval of time. Dissipation of turbulent energy contained within range of frequencies between 0 and k is given by $S_k = (S/E)\{\int_0^k F(k')dk' - [(1-\alpha)/2\alpha]kF(k)\}$ where α is found from the experimental curve $E = \text{const}\cdot t^{-2\alpha}$ and where $S/E = 2\alpha/t$. Eliminating S_k between this equation and Heisenberg's expression for S_k and using $\alpha = 0.5$, curve $F(k)$ is traced and compared with experimental measurements of Simmons and Salter. F. N. Frenkiel, USA

2433. **A. M. Yaglom, On the acceleration field in a turbulent flow** (in Russian), Doklady akad. Nauk SSSR 67, 795–798 (Aug. 1949).

Author, utilizing general theory of isotropic turbulence aided by wind-tunnel tests, develops equations suitable for study of acceleration fields in a turbulent boundary layer. Results indicate that mean square value of acceleration is proportional to mean air velocity raised to the 9/4 power. A. Petroff, USA

2434. **G. K. Batchelor, Recent developments in turbulence research**, Proc. seventh int. congr. appl. Mech. 1, 27–56 (1948).

Problem of decay of isotropic turbulence forms the primary consideration of this survey. To approach this aspect of turbulent motion, the turbulence spectrum and related energy distribution are considered. To account for the spatial character of turbulent fluctuation, a three-dimensional spectrum tensor is defined by:

$$\Gamma_i{}^j(\bar{k}) = \frac{1}{8\pi^3}\int\int\int R_i{}^j(v)e^{i\bar{k}\cdot\bar{v}}d\tau$$

where $R_i{}^j$ is the spatial correlation tensor.

The energy spectrum is obtained by integrating the corresponding energy density over the spherical surface in the vector wave number space:

$$E(k) = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi}\Gamma_i{}^i(k)k^2\sin\theta d\theta df.$$

Theory of Kolmogoroff [C. R. Acad. Sci. URSS 30, 301–305 (1941) and 32, 16–18 (1941)] regarding this energy spectrum in equilibrium decay is confirmed experimentally both for isotropic and for shear-flow turbulence. This theory implies that for k greater than those values containing most of the energy, the energy spectrum is determined only by viscosity and energy dissipation density. Hence the smallest eddies in general, and in shear flow in particular, are isotropic. Heisenberg [Zeit. Physik 124, p 628 (1948)] extends Kolmogoroff's theory by considering the form of energy exchange between wave numbers from which the energy spectrum becomes:

$$E(k) = (\epsilon/.9)^{2/3}k^{-5/3}[1 + v^3k^4/.24\epsilon]^{-4/3}$$

Rate of decay during the initial period is indicated experimentally to be inversely proportional to time. During final period this becomes inversely proportional to $t^{5/2}$. Latter result may be confirmed theoretically by assuming that inertia forces are negligible and that decay is controlled by viscosity. Initial period can be approached by a quasi-equilibrium hypothesis of Heisenberg with reference to wave numbers containing bulk of energy and smaller than those satisfying absolute equilibrium. This hypothesis is developed showing a theoretical confirmation of initial decay and for restricted turbulence Reynolds-number approximate form of energy spectrum decay.

Quasi-equilibrium cannot apply to largest eddies generating turbulence. This is due to their very large relative reaction time and to their strong dependence on continuity conditions. This situation is significant with respect to shift in decay from initial to final period. During the intermediate period largest eddies contain a dominant portion of energy and will control dissipation of energy until final period is reached. Newman A. Hall, USA

2435. **Ph. Poisson-Quinton, Theoretical and experimental research on control of the boundary layer** (in French), Proc. seventh int. Congr. appl. Mech. 2, part II, 365–381 (1948).

This lecture covers French research on boundary-layer control. In the first section, author investigates effect of one suction slot or one blowing slot on circulation of a profile. Experiments verified the theoretical result that circulation of profile increases the more the slot is moved to trailing edge. Second section deals with effect of sucking or blowing on increase of maximum lift for profiles with cambered flaps and for profiles with cracked noses. Finally, profiles of the Griffith-type were used to study influence of boundary-layer control on position of shock waves. These preliminary experiments for Mach numbers up to 0.7 gave hopeful results. Most of the theories and experiments of paper give excellent confirmation of German results before and during the war. Fritz W. Riegels, Germany

2436. **H. Görtler, Frictional resistance of a slightly corrugated plane parallel to the direction of flow** (in German), Arch. Math. 1, 450–453 (1949).

In previous work [Z. angew. Math. Mech. 25/27, 233–244 (1947); 28, 13–22 (1948); Rev 3, 132] author discussed boundary layer on a waved wall and in present communication he considers a simplified calculation of same problem. In order to reduce the complication of analysis it is assumed that (1) amplitude ϵ of wave is small compared with local boundary-layer thickness, and (2) wave length λ is sufficiently short so that development of

corresponding undisturbed boundary-layer velocity distribution over length of wave is small compared with disturbance caused by waved wall. Let y be distance from median line of waved wall measured in terms of boundary-layer thickness $\delta_0 = (\nu L/U_0)^{1/2}$ and x distance along wall measured in terms of $\lambda/2\pi$, where ν is kinematic viscosity, L plate length and U_0 free stream velocity. Differential equation to be solved is $\psi_y\psi_{xy} - \psi_x\psi_{yy} - \sigma\psi_{yyy} = -\epsilon \sin x - \frac{1}{2}\epsilon^2 \sin 2x$ with boundary conditions $\psi_x = \psi_y = 0$ for $y = 0$ and $\psi_y = 1 + \epsilon \cos x$ for $y \to \infty$. Since stream function is periodic in x at large distances from leading edge, stream function is expressed in a Fourier series

$$\psi(x, y) = \psi_0(x, y) + \tfrac{1}{2} a_0(y) + \sum_1^\infty a_n(y) \cos nx + b_n(y) \sin nx$$

Substituting into differential equation, remembering that ϵ is a small quantity, a_1 and b_1 are of order ϵ while a_0, a_2, and b_2 are of order ϵ^2. The second-order approximation for stream function is then

$$\psi(x, y) = \psi_0(y) + \epsilon[f_1(y) \cos x + g_1(y) \sin x] + \epsilon^2[f_0(y) + f_2(y) \cos 2x + g_2(y) \cos 2x]$$

where $\psi_0(y)$ is stream function of corresponding undisturbed boundary layer. Ratio of resistance of plane wall, $\bar{W}_R(\lambda;\ L)$, to that of corresponding waved wall, $\tilde{W}_R(\lambda;\ L)$, may be found from this result and obviously depends only on the nonperiodic part of above result:

$$\bar{W}_R(\lambda;\ L)/\tilde{W}_R(\lambda;\ L) = U_0'(0)/\{U_0'(0) + \epsilon^2 f_0''(0)\}.$$

Function $f_0(y)$ is determined through solution of ordinary differential equations for coefficients of Fourier series which follow from substituting in original partial differential equation. Numerical examples are discussed which show frictional resistance to be decreased by waviness.

Courtesy Mathematical Reviews F. E. Marble, USA

2437. Martin Lessen, On the stability of the free laminar boundary layer between parallel streams, Nat. adv. Comm. Aero. tech. Note 1929, 1–31 (Aug. 1949).

Author analyzes stability of free laminar boundary layer between parallel streams of an incompressible fluid using the Tollmien-Schlichting theory of small disturbances. From four possible solutions of the Orr-Sommerfeld stability equation, two which are unbounded somewhere in flow field must be excluded. Remaining two solutions are exponential in character at infinite boundaries. To obtain solution of stability equation, an expansion in powers of $(-i/\alpha R)$ is used, where R is Reynolds number and α is disturbance wave number. In its final stage, stability equation is solved by numerical integration. The eigenvalue problem (α, R and disturbance phase velocity) is solved by trial-and-error process. Author concludes that laminar boundary layer between parallel streams is an unstable-flow configuration except at low Reynolds numbers. Method of calculation of stability characteristics is successful for small absolute values of parameter $(-i/\alpha R)$. A comparison with Blasius-type flow against a flat plate shows an interesting result: For free boundary layer, instability occurs at much lower Reynolds numbers than for boundary layer against a flat plate with no pressure gradient.

M. Z. Krzywoblocki, USA

2438. H. Ludwieg and W. Tillmann, Investigation of the wall shearing stress in turbulent boundary layers (in German), Z. angew. Math. Mech. 29, 15–16 (Jan.-Feb. 1949).

Paper summarizes results given in two reports which will appear in a forthcoming issue of Ingenieur-Archiv. A method of measurement of wall shearing stress τ_w is mentioned: a small insulated metal block was built into wall level with its surface. Block is heated and loss of heat is measured. This loss is proportional to $(\tau_w)^{1/3}$ (where $\tau_w = \mu(du/dy)$).

Universal velocity law for plate flow $u/u^* = f(u^*y/\nu)$ [where $u^* = (\tau_w/\rho)^{1/2}$] is found to be valid in wall proximity for boundary layers with pressure gradient. Using this law, authors find for local drag coefficient $c_f' = [\tau_w/(\frac{1}{2})(\rho U^2)]$ in boundary layers with pressure gradient the expression: $c_f' = [\gamma^2/\gamma_0^2\ \phi]\ F_0$ where $\phi = Re \cdot \gamma/\{\gamma_0[Re \cdot \gamma/\gamma_0(\ldots)]\}$. Re is Reynolds number formed with momentum thickness θ. γ is a profile parameter which for $y = \theta$ is equal to u/U. For plate flow $\gamma = \gamma_0$ and F can be found from experiments on plate flow for which $\gamma_0 = \gamma_0(Re)$ and $c_f' = F(Re)$. For practical use following approximate relation is given: $c_f' = 0.246 \cdot 10^{-0.678H}\ Re^{-0.268}$ with $H = \delta^*/\theta$ (δ^* is displacement thickness). It follows, in agreement with experimental results, that local drag coefficient in a boundary layer with pressure drop is larger, and that obtained in a layer with pressure rise is smaller, than drag for plate flow at same Reynolds number.

Authors find that, contrary to conclusions of other investigators, friction coefficient in a boundary layer with an increasing pressure does not rise to a multiple of corresponding coefficient for plate flow.

F. N. Frenkiel, USA

2439. A. Ulrich, The plane laminar boundary layer on a cylinder (in German), Arch. Math. 2, 33–41 (1949).

Velocity distribution $U(x)$ over surface of a symmetric cylinder may be expressed in a series of odd powers, $U(x) = u_1x + u_3x^3 + u_5x^5 + u_7x^7 + \ldots$ where the u_ν are numerical coefficients which depend on the specific shape of cylindrical cross section. Boundary layer over this figure is found by solving boundary-layer equations using above free-stream velocity distribution. Result is the Blasius series for stream function,

$$\psi(x, \eta) = (\nu/u_1)^{1/2}\{u_1xf_1 + 4u_3x^3f_3 + 6x^5(u_5g_5 + (u_3^2/u_1)h_5) + 8x^7(u_7g_7 + (u_3u_5/u_1)h_7 + (u_3^3/u_1^2)k_7) + 10x^9(u_9g_9 + (u_3u_7/u_1)h_9 + (u_5^2/u_1)k_9 + (u_3^2u_5/u_1^2)j_9 + (u_3^4/u_1^3)q_9) + \ldots$$

where f_1, f_3, etc., are functions of $\eta = y(u_1/\nu)^{1/2}$ and y is distance normal to surface of cylinder. These coefficients are independent of free-stream velocity distribution and hence may be calculated generally. Function f_1 is defined by the nonlinear differential equation $f_1''' - (f')^2 + f_1f_1'' + 1 = 0$, $f_1(0) = f_1'(0) = 0$; $f_1'(\infty) = 1$, while each of the remaining functions satisfies a linear differential equation whose coefficients depend upon f_1.

(The values of $f_1(\eta)$ have been calculated by Howarth [Aeronaut. Res. Comm. Rept. no. 1632 (1935)] while later Frössling [Lunds Univ. Årsskrift (N.F.) 36, no. 4 (1940) = Fysiogr. Sällskapets Handlingar (N.F.) 51, no. 4 (1940)] improved Howarth's calculations and extended the computations to include f_3, g_3, and h_5.) In present paper author calculates g_7, h_7, k_7, g_9, h_9, k_9, j_9, and q_9 using Adams' method. The functions f_1, f_3, g_3, h_5, as well as their first and second derivatives, are tabulated to four decimal places. Functions g_7, h_7, k_7, g_9, h_9, k_9, j_9, q_9, and their first and second derivatives are tabulated to three decimal places; all tabulations cover the range $0 \leqq \eta \leqq 4.0$ in intervals of 0.1. Results allow very simple calculations of velocity or shear distributions according to Blasius series up to the terms corresponding to x^9 in the stream function.

Courtesy Mathematical Reviews F. E. Marble, USA

2440. M. Shwets, On a case of diffusion in a laminar boundary layer (in Russian), Doklady akad. Nauk SSSR 67, 799–802 (Aug. 1949).

For large concentrations of gases, Nusselt [ZAMM 10, 105–121 (1930)] showed that the component v' of velocity normal to surface absorbing or evaporating the diffusing substance does not vanish on this surface. Hence, analogy between processes of diffusion and heat transfer breaks down.

Consider a two-dimensional laminar flow under assumption that at a large distance from surface (supposed to be plane), concentration and velocity of flow are both constant, and equal to q_1' and U, respectively. According to Nusselt following condition is to be satisfied on surface if diffusion has become steady: $v' = -k[q_0'(p/p_0 - 1)]^{-1}(\partial q'/\partial y')_0$; where q' is the concentration of a gas for which surface is permeable, q_0' concentration on this surface, p_0 partial pressure of a gas, p general pressure, y' distance along normal to surface, and k coefficient of diffusion.

By introducing the dimensionless quantities $x' = lx$; $y' = ly/(Re)^{1/2}$; $v' = Uv/(Re)^{1/2}$; $u' = Uu$; $c = (q' - q_0')/(q_1' - q_0')$ into equations of motion, continuity and diffusion, and by using boundary-layer thickness δ and diffusion layer thickness δ_v, author reduces problem to integration of two partial differential equations of second order with respect to u and c. System is solved by an approximate method (see REV 3, 1544), giving expressions for u and c in terms of quantity $N' = (-k/\nu)(q_1' - q_0')[q_0'(p/p_0 - 1)]^{-1}$.

For $N' = 0$,

$$(\partial u/\partial y)_{y=0} = 0.333/x^{1/2}; \quad (\partial c/\partial y)_{y=0} = 0.333(Pr)^{1/3}/x^{1/2} \quad [1]$$

where Pr denotes Prandtl number.

Values of $(\partial u/\partial y)_{y=0}$ and $(\partial c/\partial y)_{y=0}$, given by exact solution for case $N' = 0$ [Blasius, Z. f. angew. Math. u. Phys. 56, 1–37 (1908); Pohlhausen, ZAMM 1, 252–268 (1921)], and those obtained by approximate solution [1] differ only by the factor 0.332. E. Leimanis, Canada

2441. **R. Boucher and P. Montigny, Influence of the constructive factor ψ_0 on the yield curve of a cyclone centrifuge** (in French), C. R. Acad. Sci. Paris 229, no. 22, 1126–1128 (1949).

2442. **A. M. Obukhov, The structure of the temperature field in a turbulent flow** (in Russian), Izvestiya Ser. Geograf. Geofiz. 13, 58–69 (1949).

Kolmogoroff [Doklady Akad. Nauk SSSR 30, 301–305 (1941)] defined the notion of locally isotropic turbulence and introduced the structure function of the field of velocities of turbulent flow. Author introduces the structure function of the temperature field of turbulent flow, defined as mean value of the square of the difference of temperature at two points M and M':

$$H(M, M') = \overline{[T(M') - T(M)]^2}.$$

In case of local isotropy and very large Reynolds numbers, the function $H(M, M')$ depends only on distance r between points M and M', it being understood that both points belong to same domain and distance r is small in comparison with scale of turbulence. Using method similar to that applied by Kolmogoroff in the case of locally isotropic field of velocities of a turbulent flow, author is led to some similar results concerning structure function of the temperature field. For instance, if points M and M' are "not very close together" it is found that $H(r)$ is of the form

$$H(r) = k^2 \bar{N} \bar{\epsilon}^{1/3} r^{2/3} \quad [1]$$

where $\bar{N}$ is the mean velocity of homogenization of the temperature field, $\bar{\epsilon}$ is mean dissipation of energy per unit of mass, and k a numerical constant. The dependence upon $r^{2/3}$ is the same result as that obtained by Kolmogoroff and Obukhov [title source, 453–466 (1941)] for the field of velocities. For "small" values of r following approximate formula [2] $H(r) \approx 1/3\, \bar{N}\lambda^{-1}r^2$ holds, where λ is thermometric conductivity.

A scale r_1 is assigned to the characteristic of field of pulsations of temperature field. It is defined geometrically as the value of the abscissa of point of intersection of the two asymptotic representations [1] and [2] of $H(r)$ for "large" and "small" values of r respectively: $r_1 = (27k^6\lambda^5/\bar{\epsilon})^{1/4}$.

Since Prandtl's number $Pr = \nu/\lambda$ is of the order of unity for atmosphere, r_1 is of the same order of magnitude as the inner scale of turbulence (size of smallest eddies) $\eta \sim (\nu^3/\bar{\epsilon})^{1/4}$, introduced by Kolmogoroff (op. cit.).

Throughout paper the case of fully developed turbulence of dynamical origin is considered, and pulsations of temperature are assumed to be so small (in comparison with the mean absolute value of temperature of the medium) that they do not influence sensitively the regime of turbulent pulsations of velocity of flow. Radiative heat transfer and heating due to dissipation of mechanical energy of turbulence are neglected.

In formula [24] of text and in preceding formula there are misprints. E. Leimanis, Canada

2443. **A. M. Yaglom, On the local structure of the temperature field in a turbulent fluid** (in Russian), Doklady Akad. Nauk SSSR 69, 743–746 (Dec. 1949).

Recently Obukhov [see preceding review], using method of similarity and dimensional considerations, investigated local structure of temperature field in a turbulent flow of an incompressible viscous fluid at very large Reynolds numbers. Author improves some of these results by direct consideration of equations of hydrodynamics. Under assumptions made by Obukhov concerning nature of flow, field of velocities satisfies usual equations of motion of a viscous incompressible fluid, while the temperature field satisfies equation

$$\partial T/\partial t + \Sigma_{j=1}^{3} v_j \partial T/\partial x_j = \lambda \Delta T \quad [1]$$

where λ denotes thermometric conductivity. For very large Reynolds numbers turbulent motion may be considered as locally homogeneous and locally isotropic [Kolmogoroff, title source, 30, 301–395 (1941)]. Under this assumption author deduces the relation

$$D_{lTT}(r) - 2\lambda d D_{TT}(r)/dr = -\frac{4}{3}\bar{N}r \quad [2]$$

connecting the struction functions

$$D_{lTT}(r) = \overline{[v_l(M') - v_l(M)]\,[T(M') - T(M)]^2}$$

$$D_{TT}(r) = \overline{[T(M') - T(M)]^2}$$

and the constant

$$\bar{N} = \frac{3}{2}\lambda\, D'_{TT}(0) \quad [3]$$

introduced by Obukhov (op. cit.). [For a corresponding relation between the structure functions $D_{ll}(r)$ and $D_{lll}(r)$ of the field of velocities, cf. Kolmogoroff, title source, 32, 16–18 (1941)]. For small or large values of r it follows from [2] that

$$D_{TT}(r) \approx \frac{1}{3}\bar{N}\,\lambda^{-1} r^2 \quad [4]$$

or

$$D_{lTT}(r) \approx \frac{4}{3}\bar{N}r \quad [5]$$

respectively.

If F denotes the dimensionless quantity

$$D_{llT}(r) = D_{TT}(r)[D_{ll}(r)]^{1/2} \qquad [6]$$

which for large Reynolds numbers and large values of r may be considered as a negative constant, it follows from [5] and [6] that

$$D_{TT}(r) = -\frac{4}{3}F^{-1}Nr[D_{ll}(r)]^{1/2} \qquad [7]$$

Using the "two-thirds law" for the field of velocities:

$$D_{ll}(r) = C\epsilon^{2/3}r^{2/3}, C = (-4/5S)^{2/3}, S = D_{lll}(r)/[D_{ll}(r)]^{3/2}$$

author obtains from [7] the "two-thirds law" for temperature field (given previously by Obukhov):

$$D_{TT}(r) = k^2 N \epsilon^{-1/3} r^{2/3}, k^2 = -\frac{4}{3}F^{-1}(-5S/4)^{2/3} \qquad [8]$$

The dimensionless constant k^2 used by Obukhov is now expressed in terms of S and F which have simple probability-theoretical meaning. Finally, an interpolation curve for $D_{TT}(r)$ is given which reduces to the asymptotic formulas [4] and [8] for their corresponding ranges of validity. E. Leimanis, Canada

2444. L. S. G. Kovasznay, Hot-wire investigation of the wake behind cylinders at low Reynolds numbers, Proc. roy. Soc. Lond. Ser A 198, 174–190 (Aug. 15, 1949).

Hot-wire measurements presented here deal with vortex-street pattern behind circular cylinders. Periodic fluctuations in wake are found to appear at a Reynolds number of 40, and their dimensionless frequency (Strouhal number) observed to increase in the Reynolds number range of 40–1000. Mean-velocity measurements across wake are made at several distances behind cylinder at Reynolds numbers slightly below and slightly above critical value of 40, indicating considerable decrease in wake width due to the vortex street. Magnitude of fluctuations is also determined; fluctuations of maximum intensity (14% of mean velocity) are found to appear 7 diam downstream from cylinder, while practically no fluctuations were observed in immediate vicinity of cylinder. This indicates that "the vortex street can be considered as an instability of the laminar wake, that develops up to an amplitude limit, but dies out before becoming turbulent."
Andrew Fejer, USA

2445. Dale L. Burrows, Albert L. Braslow, and Neal Tetervin, Experimental and theoretical studies of area suction for the control of the laminar boundary layer on a porous bronze NACA 64A010 airfoil, Nat. adv. Comm. Aero. tech. Note 1905, 53 pp. (1949).

An exploratory wind-tunnel investigation was made on an NACA 64A010 airfoil with central-section surface being composed of porous sintered bronze. Through area suction, full-chord laminar flow was achieved to Reynolds numbers of about 8×10^6. Results indicated that extension of low-drag characteristics to higher Reynolds numbers might be possible with controlled chordwise distribution of suction inflow. More information is needed regarding boundary-layer stability and surface irregularities under suction action, in order to better correlate the experiments and elementary theory presented. R. G. Folsom, USA

2446. K. Wieghardt, On diffusion phenomena in turbulent boundary layers (in German), Z. angew. Math. Mech. 28, 346–355 (Nov.-Dec. 1948).

Paper discusses measurement of temperature in an air flow over a flat plane, due to a point and line source of heat lying in plane. Results of this turbulent heat transfer are used to derive empirical formulas for temperature distribution at different distances from source. Reynolds number of the tests, based on free-stream velocity and distance from leading edge of plate, varied from 0.9×10^6 to 5×10^6. Formulas are stated to be not directly applicable to meteorological problems since, apart from differences in Reynolds number, scale and intensity of turbulence in wind tunnel will be very different from that in free atmosphere.

Results are compared with theory of Frost [Proc. roy. Soc. A 189, p. 20 (1946)] and differences in exponents of certain terms in temperature formula are found. G. M. Lilley, England

2447. E. Brun and Marcel Vasseur, Contribution to the thermic study of the laminar boundary layer (in French), Proc. seventh int. Congr. appl. Mech. 2, part I, 249–263 (1948).

Paper considers theoretical problem of integrating laminar boundary-layer equations for a two-dimensional flow as affected by heat flow. In a sense it represents an extension of work of Roy, and of Brainerd and Emmons. Analysis is based on usual equations of continuity, momentum, and energy for a viscous compressible fluid. Only a first approximation is considered, in which physical characteristics of fluid are treated as constants within boundary layer. Two cases are treated: that of no heat flow and that of constant temperature at the solid boundary.

Application is made to case of a symmetric wedge along which external flow is an exponential function of distance. Coefficients of friction and convection, or equivalent displacement thicknesses, are calculated. A method of applying procedure to a profile of any shape is then developed. No numerical results are given.
M. J. Thompson, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 2285, 2394, 2403, 2404, 2414, 2419, 2465

2448. Leonard Sternfield and Ordway B. Gates, Jr., A theoretical analysis of the effect of time lag in an automatic stabilization system on the lateral oscillatory stability of an airplane, Nat. adv. Comm. Aero. tech. Note 2005, 29 pp. (Jan. 1950).

This investigation of the effect of time lag on the lateral oscillations of an airplane consists of two parts: (a) Determination of time lag which would result in neutral stability, and (b) determination of loss in stability produced by a given time lag. Part (a), based on the frequency response method, is solved in a relatively simple manner. Part (b) is solved by means of an analytical-graphical procedure. Simplified alternate methods are described and results calculated are compared with those obtained from step-by-step calculations. Method is applied to a typical airplane having rudder controlled by an auto-pilot sensitive to yawing acceleration. John R. Spreiter, USA

2449. John DeYoung, Spanwise loading for wings and control surfaces of low aspect ratio, Nat. adv. Comm. Aero. tech. Note 2011, 36 pp. (Jan. 1950).

Low-aspect-ratio wing theory of R. T. Jones has been extended to provide a means of predicting aerodynamic characteristics resulting from deflection of arbitrary control surfaces of low-aspect-ratio on low-aspect-ratio wings. Results are applicable to low-aspect-ratio wings at all Mach numbers and to moderate-aspect-ratio wings traveling at near sonic speeds. Considered wing plan forms are arbitrary except that no part of trailing edge may extend ahead of the most forward line of maximum span.

Results of numerous applications to both symmetric and antisymmetric control deflections are given in a series of charts. Report also contains some comparisons between lift and damping in

roll predicted by low-aspect-ratio wing theory and that predicted by supersonic linearized lifting-surface theory and by application of Weissinger's subsonic lifting-line theory. These comparisons should be of interest to anyone attempting to determine useful limits of low-aspect-ratio wing theory.

John R. Spreiter, USA

2450. **Herbert S. Ribner and Frank S. Malvestuto, Jr., Stability derivatives of triangular wings at supersonic speeds,** Nat. adv. Comm. Aero. Rept. 908, 9 pp. (1948).

Analysis of stability derivatives of low-aspect-ratio triangular wings at subsonic and supersonic speeds, given in NACA T.N. no. 1423, is extended to apply to triangular wings having large vertex angles and traveling at supersonic speeds. Lift, rolling moment due to sideslip, and damping in roll and pitch for this more general case have been treated elsewhere on basis of the theory of small disturbances. Surface potentials for angle of attack and rolling taken therefrom are used to obtain the several side-force and yawing-moment derivatives that depend on leading-edge suction, as well as a tentative value for rolling moment due to yawing. Lift and moment due to downward acceleration are obtained on basis of an unpublished unsteady-flow solution. All known stability derivatives of triangular wing at supersonic speeds, regardless of source, are summarized for convenience and presented with respect to both body axes and stability axes. Results are limited to Mach numbers for which the triangular wing is contained within Mach cone from its vertex. Spanwise variation of Mach number in case of yawing is neglected, although effect must be of importance. From authors' summary by M. A. Heaslet, USA

2451. **Doris Cohen, Theoretical loading at supersonic speeds of flat swept-back wings with interacting trailing and leading edges,** Nat. adv. Comm. Aero. tech. Note 1991, 55 pp. (Dec. 1949).

In previous work [NACA T.N. 1555 (1948)] author used Lagerstrom's method of superposed, linearized, conical flows [same source, 1685 (1948)] to calculate loading on thin, flat, sweptback wings of rectilinear plan form in supersonic flow, subject to limitation that Mach cone from trailing-edge apex did not cut leading edges ("noninteracting" leading and trailing edges). Present paper is directed toward an analysis of wings that violates this assumption, such as those having large sweep or aspect ratio, or moderate sweep and aspect ratio, at Mach numbers close to unity. Taking the starboard wing, let A denote intersection of the trailing edge, apex (downstream) Mach cone with the leading edge, B the intersection of the Mach cone from A with the trailing edge, C the intersection of the Mach cone from B with the leading edge, etc. Superposing conical flows, the exact loading is determined in ABC, while a quasi-exact solution is determined in BCD, subject only to a small, estimated correction to satisfy the Kutta condition. Beyond BCD, use of conical flows becomes so complex as to be of no practical use, but author shows that simple sweep theory (i.e., neglecting the influence of center section of wing) is sufficiently accurate in such regions. Numerical calculations are given for loading on streamwise sections in ABC and BCD for two different plan forms, and results are compared with simple sweep theory, slender-wing theory [same source, 1824 (1949)], and a modified, two-dimensional theory proposed by author. The last approximation, which is based on subsonic flow, normal to its surface, about a flat plate, which is coincident with a wing section *normal* to stream, proves to be sufficiently accurate for practical purposes. Loading in the zone of action of a wing tip lying entirely inside the Mach cone from trailing-edge apex Mach cone is calculated approximately on the assumption of cylindrical flow outboard of tip. An appendix indicating a conical flow calculation for the tip is included, but author argues that resulting complexity is not warranted in view of known large deviations from ideal flow in this region. Hence, within limitations imposed by assumption of linearized inviscid flow, author succeeds in developing a practical method for calculation of loading on all supersonic, rectilinear, sweptback plan forms of practical interest except those for which effective span $[(M^2 - 1)^{1/2}$ times geometric span] is less than tip chord, and she points out that slender wing theory is applicable to latter class. J. W. Miles, USA

2452. **Z. Ruzha, Dynamic testing device for landing gears which simulates ground unevenness** (in French), Proc. seventh int. Congr. appl. Mech. 2, part II, 525–532 (1948).

To simulate shocks imposed upon aircraft landing gear by unevenness of landing field, a rotating drum with a sliding obstruction is employed. Speed of drum and height of obstruction may be varied to obtain data regarding shock-absorbing qualities and resonance of landing gear for a number of likely conditions. Rig is described; several illustrations show details; theoretical considerations are discussed; and results of a few tests evaluated.

Frederick K. Teichmann, USA

2453. **J. E. Wignot and Frederic M. Hoblit, Landing-gear oscillations due to unstable skidding friction,** J. aero. Sci. 16, 491–495 (Aug. 1949).

Elementary theory shows that self-excited lateral vibration may exist in an airplane landing gear with brakes applied and wheels skidding. Severity of vibration depends on negative slope of friction-skidding velocity curve. Influence of mass, stiffness and natural damping of gear is considered in relation to amplitude, load, and stresses produced.

R. N. Arnold, Scotland

2454. **F. B. Gustafson, Kenneth B. Amer, C. R. Haig, and J. P. Reeder, Longitudinal flying qualities of several single-rotor helicopters in forward flight,** Nat. adv. Comm. Aero. tech. Note 1983, 25 pp. (1949).

Continuing an earlier paper (see REV 2, 916), authors present flight-test measurements and corresponding pilot's opinions of three single-rotor helicopters. Helicopter A is a conventional helicopter with torque-counteracting tail rotor; helicopter C has the same general arrangement but is equipped with a gyroscopic device for improving stability and control; and helicopter B is the same as A with a small fixed horizontal tail surface added.

Stick motion from trim, pitching velocity, and normal acceleration in a "pull and hold" maneuver is measured at level flight at cruising speed of approximately 80 mph. A theoretical analysis of this maneuver is also made for helicopters A and B, but no details of it are given.

Three conclusions are drawn: (1) Most important consideration is prevention of prolonged stick-fixed divergent tendencies. (2) When this divergence is eliminated, additional improvement is concluded to relate to a continuous development of normal acceleration in contrast to a pause during the first second following abrupt control deflection. (3) Normal acceleration characteristics can be theoretically predicted. These conclusions are also expressed in the form of tentative flying-qualities requirements. Heinz Parkus, Austria

2455. **F. W. J. Olver, Transformation of certain series occurring in aerodynamic interference calculation,** Quart. J. Mech. appl. Math. II, 452–459 (Dec. 1949).

Calculation of aerodynamic interference on wings in rectangular wind tunnels involves laborious computation of sum of a slowly convergent double series. Object of paper is to establish a

transformation converting this series into a rapidly convergent and easily computable form. Numerical values of new series are given for cases corresponding to wind tunnels having side ratios of 1:1 and 2:1. General forms of slowly convergent series which can be similarly transformed are also stated. Paper is followed by a note by G. E. H. Reuter in which another proof of transformation is given. John R. Spreiter, USA

2456. **W. D. Brown, The effect of tension on the porosity of a parachute fabric,** Rep. memo. aero. Res. Counc. Lond. 2325, 10 pp. (1949).

Author reports test made for purpose of setting up recommendations for selection of optimum instrument for measuring porosity in parachute cloth under operational load conditions. A variety of fabrics and weaves were tested. Facts are that operational stresses affect porosity (amount of air passed per unit area for a fixed pressure head), which in turn affects aerodynamic performance of parachute. Data are presented in graphical form. M. G. Scherberg, USA

2457. **A. J. Marx and J. Buhrman, The effect of a spring tab elevator on the static longitudinal stability of an aeroplane,** Nat. Luchtlab. Amsterdam Rep. 1547, 15, 3 pp. (1949).

A theoretical investigation is made of rate of change of pitching moment with incidence (interpreted here as static margin). Stick-fixed stability is found to decrease unless $b_1 > 0$ (elevator trails against wind). Influence of spring tab on stick-free stability is found to be negligible. A. D. Young, England

2458. **Alex Goodman and Glenn H. Adair, Estimation of the damping in roll of wings through the normal flight range of lift coefficient,** Nat. adv. Comm. Aero. tech. Note 1924, 31 pp. (1949).

Methods are advanced for predicting damping in roll for wings operating at medium- and high-lift coefficients where potential-flow methods of estimating c_{l_p} are not sufficiently accurate. Starting with potential-flow solutions at zero-lift coefficient, empirical corrections, based on experimental variations of lift with angle of attack and drag, are developed.

Three methods are considered: First employs a lift-curve-slope correction only; second adds a correction for drag as a whole; and third breaks drag correction down into its induced and profile-drag components. A comparison of these methods with experimental results shows that method based on lift-curve-slope correction alone is nearly as reliable as the other two. Thus it is indicated that drag correction is quite secondary to lift correction.

Formulas giving a correction for section-lift-curve slope and for compressibility are also included. Arthur L. Jones, USA

2459. **E. T. Binckley and Jack Funk, A flight investigation of the effects of compressibility on applied gust loads,** Nat. adv. Comm. Aero. tech. Note 1937, 24 pp. (Aug. 1949).

Two jet-propelled airplanes were flown at different speeds in rough air to investigate effects of compressible flow on loads caused by turbulence. Data were obtained at Mach numbers of 0.28, 0.62, and 0.68 for gust velocities up to 15 fps. A comparison of load experience at various speeds indicated that compressibility did not affect total gust loads on airplanes in turbulent air over the range of test Mach numbers. It is important to note that results are restricted to test airplanes, and additional research is needed before they can be generally applied.

Statistical methods were employed for comparing gust experience on assumption that successive passes of airplane gave same gust distributions. Under test conditions this seems to be a valid assumption. Technique may be of interest to those who are contemplating studies where statistical comparison of aerodynamic data is necessary. Philip Donely, USA

2460. **Sydney M. Harmon, Stability derivatives at supersonic speeds of thin rectangular wings with diagonals ahead of tip Mach lines,** Nat. adv. Comm. Aero. Rep. 925, 14 pp. (1949).

See REV 2, 96.

2461. **Benjamin Smilg, The instability of pitching oscillations of an airfoil in subsonic incompressible potential flow,** J. aero. Sci. 16, 691–696 (Nov. 1949).

It is shown that undamped oscillations in only one degree-of-freedom (pitch) can occur under the condition that moment of inertia of airfoil is large and axis of rotation is located forward of quarter-chord, but not too far forward of leading edge of airfoil.

A graph is includeds howing the combinations of inertia parameter, location of rotational axis, and a nondimensional stiffness parameter, for which damping of oscillations is zero. Study of results shows that this single degree-of-freedom mode of undamped oscillations will not occur in most wing or tail flutter problems for conventional airplanes. Only in exceptional cases—e.g., sweptback wing with extremely heavy weights near tips—can conditions required for flutter be satisfied.

Application of theory to problems involving airplane stability is discussed briefly. It is concluded that, although undamped oscillations of this type are not likely to be important in stability problems, the pseudostatic aerodynamic forces and moments should not be used in dynamic stability analysis if an accurate estimation of damping of oscillations is desired.

J. Buhrman, Holland

2462. **P. Höhler and I. von Köppen, Experimental flights for testing of a reactor as an expedient for the termination of dangerous spins,** Nat. adv. Comm. Aero. tech. Memo. 1221, 1-18 (July 1949).

After an F.W. 56 plane attained steady spin conditions, known forces were applied at rear, along longitudinal axis or lateral axis, for a time sufficient to let new steady spin conditions appear. The three angular velocity components, the three acceleration components, and the rate of descent were recorded in initial and final conditions. Although control setting and engine speed did not change during tests, they were recorded also. Numerical values of the C_M and C_N coefficients, corresponding to pitching and yawing moments applied by the reactor, were calculated. Relations between moment coefficients and changes in values of geometrical or dynamical characteristics of spin are given. In reviewer's opinion, this is one of the rare experimental works where such relations are proposed. It has been established that, with tested plane, a diving moment altered spin conditions in a manner that did not facilitate spin termination. On the contrary, a yawing moment, opposed to rotation, seemed able to lead to desired result. F. C. Haus, Belgium

2463. **Robert C. Seamans, Jr., Benjamin G. Bromberg, and L. E. Payne, Application of the performance operator to aircraft automatic control,** J. aero. Sci. 15, 535–555 (Sept. 1948).

Paper presents an analysis technique for design of an automatic aircraft-control system. Authors make use of performance operator which is very similar to transfer function found in servomechanism work, and which presents dynamic behavior of a component or group of components as a function of the frequency of a steady-state sinusoidal input or disturbance to component. The performance operators of several components in a proposed control loop may be readily combined in accordance with prin-

uple of servomechanisms to give an over-all performance operator for complete system.

Three steps are presented by which nature of projected control system can be predicted from over-all performance operator. First, a peak in amplitude ratio curve of magnitude greater than 1.3 indicates poor system damping, and leading phase angles at high frequency indicates instability. Second, the open-loop performance operator may be plotted as a Nyquist diagram and system stability proved or disproved. Third, step-function response of system may be obtained (by suitable approximations to be described) and from inspection of time history, a response time and stability coefficient may be found. Authors define response time as time at which system output reaches and never exceeds a value within 5% of final steady state value. Stability coefficient is defined as one minus the ratio of successive oscillatory peaks.

In order to find the system step-function response, authors consider a square wave input function whose half period is roughly equal to response time of system, and decompose the square wave into a Fourier series. Performance operator is then applied to the series term by term, yielding a new series expressing output response as a function of time. For engineering purposes summation is carried to the tenth term. By an inverse process, a flight test method is presented by which performance operator of an airplane is derived from its measured step-function response. Authors also describe an electrical device for summing nine odd harmonics and recording the result oscillographically.

Clifford L. Muzzey, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 2195, 2285)

2464. **R. Weber and W. Ruppel, Flutter study at supersonic speeds I: Flutter with one degree of freedom (plane problem)** (in French), Off. nat. Étud. Rech. áero. Rep. 35, 32 pp. (1949).

Adopting aerodynamic forces as determined and tabulated by theory of Borbely and Schwarz [same source, 5 (1948); tables not yet published], extensive flutter calculations have been carried out for the "two-dimensional" case of a rigid wing, free to oscillate under elastic restraint Φ, or $\Phi(1 + i\alpha)$ (α —coefficient of structural damping) about a spanwise axis at distance ξ (in terms of chord) behind leading edge, in a supersonic flow ($M \geq 1.1$). Results cover almost the whole range of variation of (dimensionless) structural parameters ξ, Φ, α, θ (mass ratio), and of aerodynamic parameters M (Mach number) and ω_r (reduced frequency). Important conclusions are that the kind of one-degree-of-freedom flutter considered is possible only for $M < 1.58$ and for $-1.8 < \xi < 0.65$. Structural damping has a considerable stabilizing influence. Mass balance is not suitable for reduction of flutter. Results apply also, as stated in report, to case of a control surface free to oscillate about a hinge connecting it to a wing, rigidly fixed in space.

J. H. Greidanus, Holland

2465. **Samuel Pines, A unit solution for the load distribution of a nonrigid wing by matrix methods,** J. aero. Sci. 16, 470–476 Aug. 1949).

A matrix method for determining spanwise lift distribution for a flexible unswept wing is developed. Aerodynamic terms are derived by Multhopp method; elastic terms are expressed in terms of influence coefficients. For symmetrical case, angle of attack at root is taken as unity; for unsymmetrical case, aileron deflection is taken as unity. Following certain linearizing assumptions regarding aerodynamic coefficients, matrix equations are developed for lift distributions corresponding to symmetrical and unsymmetrical cases as explicit functions of dynamic pressure, load factor, and rate of roll. An iterative procedure for solving these equations for lift distributions corresponding to these parameters is described. Analysis is then extended by use of matrix techniques so that, with only a relatively small additional amount of computation, critical speeds for wing divergence, aileron reversal, and wing-aileron divergence may be obtained as well as such basic flight criteria as the wing lift slope, induced drag, and rate of roll. A numerical example is carried out to illustrate method. Possibility of extending procedure to case of elastic swept wing is mentioned.

Benjamin Smilg, USA

2466. **Alexander Mendelson, Aerodynamic hysteresis as a factor in critical flutter speed of compressor blades at stalling conditions,** J. aero. Sci. 16, 645–652 (Nov. 1949).

It has been established experimentally that flutter speed of a wing bending vs. wing torsion system is reduced considerably as the mean operating angle-of-attack of wing becomes large and enters stall region. Author shows that this behavior can be anticipated by flutter calculations which employ a simple modification of aerodynamic forces obtained from conventional oscillating airfoil theory (linearized theory for an inviscid fluid). Modification consists of introducing an empirically determined phase lag angle between displacement and associated theoretical air forces. This lagging phase angle is meant to account for the separation phenomena which are present in the system, a concept which was originally used by B. Smilg [J. aero. Sci., 16, no. 11 (Nov. 1949)] in discussing transonic flutter problems.

Author discusses in detail the flutter of a particular wing for which experimental data are available. It is shown that excellent correlation is obtained between results of calculation and experiment when empirical phase lag angle is determined by setting its value, for a particular wing mean angle-of-attack, proportional to the amount $[C_{L\alpha} - C_{L'\alpha}]$; here $C_{L'\alpha}$ is slope of lift coefficient vs. angle-of-attack curve at angle-of-attack in question, while $C_{L\alpha}$ is the slope for low angles-of-attack where separation does not occur.

Martin Goland, USA

2467. **R. Mazet, Application of the method of "accompanying effect" in the determination of the critical velocity of flight and of the degree of explosivity of flutter** (in French), Proc. seventh int. Congr. appl. Mech. 4, 96–108 (1948).

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 2217, 2346, 2356, 2366, 2415)

2468. **John L. Crigler, Application of Theodorsen's theory to propeller design,** Nat. adv. Comm. Aero. Rep. 924, 17 pp. (1949).

Paper contains a collection of fundamental formulas of Theodorsen's theory and presents graphs illustrating distribution of circulation under optimum condition, efficiency, mass coefficient, axial energy loss factor, and axial loss ratio for single- and dual-rotating propellers with various numbers of blades. Some of graphs have been recalculated or are presented for the first time; others are reproduced from works by other authors. Some deviations (fig. 4, a,b,c) from corresponding graphs in Theodorsen's book "Theory of Propellers" have not been explained to reader.

Einer Hogner, Sweden

2469. **Sumner Alpert, Design method for two-dimensional channels for compressible flow with application to high-solidity cascades,** Nat. adv. Comm. Aero. tech. Note 1931, 1–33 (Aug. 1949).

Recent activity in field of axial-flow compressors has made subject under review an important one. While much work has been

done on flow of incompressible fluids through cascades, this is one of few papers on flow of compressible fluids. Method is an approximate numerical one in which computer starts with an assumed curve for suction side of a cascade vane and velocity distribution along this curve, and, step by step, draws successive streamlines. At any time, one of streamlines may be chosen as pressure side of cascade vane, thus completing vane contour. Mass flow between proposed streamlines and curvature of prescribed boundary is noted. This leads to an integral taken normally to streamline involving streamline curvature. Assumptions of constant curvature and vertex flow are both investigated, and differences which result are shown in an example. There is no discussion as to agreement of this method with that of general differential equation, nor is there any indication given to show that resulting flow upstream and downstream of cascade might be considered reasonable in a case of practical interest.

Howard W. Emmons, USA

2470. Shao-Pan Liang, Performance analysis of centrifugal compressors, J. aero. Sci. 16, 435–437 (July 1949).

Introduction of thermodynamic equations into Euler's formulas, and elimination of certain friction terms give a simplified relation for pressure quantity characteristic of centrifugal compressor. Author compares experimental data and calculated curves which show reasonable agreement in neighborhood of best efficiency. H. H. Anderson, Scotland

2471. Frank L. Wattendorf, Factors influencing the utilization of high mass flow in axial flow compressors, Proc. seventh int. Congr. appl. Mech. 2, part II, 398–411(1948).

High thrust per unit frontal area of a jet engine can be achieved by increasing the air-mass flow per unit area of the compressor. Paper discusses design limitations of axial-flow compressor for high mass flow. In particular, relationship between stage pressure ratio, or temperature rise, and mass flow is developed for several design methods using simplifying assumptions. Influence of hub-tip ratio and inlet axial-flow Mach number on temperature rise and mass flow is shown for several cases. Study is limited to a single stage and represents a qualitative method of comparing compressor types.

Analysis shows that a low hub-tip ratio and high axial Mach number results in a high mass-flow ratio. For compressor stage with symmetric or nearly symmetric velocity vector diagram, however, high axial Mach number results in low pressure ratio per stage. A comparison of free-vortex and rigid-body rotation designs is made on basis of relationship between ideal stage-temperature rise and mass flow ratio for different values of axial Mach numbers. High values of mass flow require high axial inlet Mach numbers, and for same temperature rise rigid-body design gives higher mass flow ratios than free-vortex design.

Paper leads to conclusion that for the all-important increase of mass flow per unit frontal area, more favorable combinations of mass flow and pressure ratio per stage are attainable by deviating from limiting assumptions, such as constant work and rigid-body rotation. Several new approaches are possible to increase mass-flow ratio of axial-flow compressors, and may bring solutions to problem of increasing thrust per frontal area of turbojet engines. For example, flows with variable temperature rise from hub to tip appear to have more favorable mass-flow characteristics than either free-vortex or solid-body flow. H. E. Sheets, USA

2472. I. M. Davidson, The axial compressor blade fouling problem, Proc. seventh int. Congr. appl. Mech. 2, part I, 200–211 (1948).

Paper describes tests of multistage axial-flow compressors conducted in order to determine losses in performance caused by accretion of an oily sludge on blades. It was found that losses in efficiency of about 5 per cent and losses in pressure ratio of about 7 per cent occurred after 100 hr of operation. Losses of this magnitude would disqualify gas turbines for such industrial applications as electrical power stations and railway locomotives. Problem is not of particular concern in aircraft engines because, in this case, relatively clean air is used and frequent inspection and cleaning of engine is the rule.

Author considers various possible processes for air filtration. Orthodox filters are not suitable because of extremely small size of particles (of the order of 1μ in diam) which contribute to fouling of blades. It is concluded that the most suitable filter would probably consist of a scrubbing chamber in series with a centrifugal separator. Brief consideration is given to possibility of reducing fouling losses by suitable blade design. It is suggested that losses can be minimized by reducing curvature of upper trailing surface of blades. John V. Becker, USA

2473. Donald L. Mordell, Contraflow contrarotating turbo-compressors, Canad. J. Res. Sec. F, 7, 285–298 (1949).

Paper discusses possibilities of an axial-flow turbocompressor consisting of n mechanically disconnected (free-wheeling) stages. Each of n stages consists of a turbine-blade row and a compressor-blade row arranged concentrically on same wheel, and each wheel rotates in opposite direction to its neighbors. Axial velocity of working fluid through turbine stages is opposite to that in compressor stages.

By neglecting radial variations of velocities, and using velocity triangles at inlet and outlet from a blade row, a difference equation is derived relating the speed of a rotor to speed of the two adjacent rotors. This equation must then be solved subject to two boundary conditions defined by direction of working fluid at entry to compressor at one end, and to turbine at other end. Contraflow character of machine thus locates these boundary conditions at opposite ends. Author circumvents difficulties of solution by using simplifying assumptions which permit a simple solution without entirely invalidating the conclusions.

Calculations were made of efficiency and compression temperature rise which were compared with results from an experimental compressor. No details given of experimental compressor or of techniques of measurement. Data show agreement of theory and experiment within 10%. D. A. Morelli, USA

2474. J. Howard Childs, Richard J. McCafferty, and Oakley W. Surine, Effect of combustor-inlet conditions on performance of an annular turbojet combustor, Nat. adv. Comm. Aero. Rep. 881, 25 pp. (1947).

Combustion performance and altitude operational limits of an annular combustion chamber of a turbojet engine were studied by operating combustor over a range of conditions of air flow, inlet pressure, inlet temperature, and fuel flow. Decreasing inlet temperature and pressure and increasing inlet velocity resulted in unfavorable changes of combustor performance, such as resonant combustion, a lower combustion efficiency, and appearance of a maximum in curve of combustor temperature rise against fuel-air ratio. Inlet conditions of combustor simulate combinations of engine speed and altitude. Above a certain altitude a region in engine-speed range occurs, at which maximum obtainable temperature rise in combustor is insufficient for keeping turbojet engine, using that combustor, running. This is due to unfavorable combination of inlet velocity, temperature, and pressure simulating that height and speed. Combustion efficiency is low with these conditions. Measured temperature distribution at combustor outlet is rather irregular. Danger of vapor-lock in burners is

mentioned. Results of measurements are presented in graphs and tables. J. G. Slotboom, Holland

2475. **Arthur W. Goldstein, Sumner Alpert, William Beede, and Karl Kovach, Analysis of performance of jet engine from characteristics of components. II—Interaction of components as determined from engine operation,** Nat. adv. Comm. Aero. Rep. 928, 15 pp. (1949).

See REV 1, 1406.

2476. **Newell D. Sanders and Michail Behun, Generalization of turbojet engine performance in terms of pumping characteristics,** Nat. adv. Comm. Aero. tech. Note 1927, 33 pp. (Aug. 1949).

Characteristics of a basic turbojet engine consisting of compressor, combustor, and turbine can be presented in terms of pumping characteristics, i.e., corrected air flow, ratio of engine-outlet to -inlet total pressure, ratio of engine-outlet to -inlet temperature, Reynolds number index, corrected engine speed, and corrected fuel-air ratio. Such a presentation offers a general method for describing engine independently of characteristics of other elements of propulsion system. This method also permits rapid estimation of performance of complex propulsion systems involving basic turbojet engine.

Pumping characteristics are illustrated by experimentally determined characteristics of a typical basic turbojet engine. As an illustration of utility of this method of presenting engine characteristics, effects of inlet-pressure losses, inlet temperature and altitude and heat additions in tailpipe on performance of propulsive systems are estimated.

From author's summary by J. G. Slotboom, Holland

2477. **Raymond Siestrunck and Jean-J. Bernard, Aerodynamics of coaxial contrarotating systems** (in French), Off. nat. Étud. Rech. aéro. Rep. 36, 20 pp. + 6 tables (1949).

Authors generalize methods, developed earlier by them for single screws, to a system of two coaxial screws of preferably opposite rotation. Determining distribution of circulation along blades in a special example, they find comparatively good agreement with results obtained by using Lock method. Optimum conditions for the system are derived and discussed in relation to corresponding condition for a single screw. General results for 3- and 4-bladed screws are given in five diagrams and special results relating to an illustrating example in a sixth.

Interaction between screws is considered steady in relation to each of them by approximating the pulsating action of the other by its mean value. In a known way problem is transformed to two-dimensional problems of Goldstein equations with boundary conditions. For numerical evaluations, authors frequently use a known electrical analogy device representing flow around screws.

Some typographical errors are apparent in paper.

Einar Hogner, Sweden

Flow and Flight Test Techniques

(See also Revs. 2446, 2455, 2521)

2478. **A. D. S. Carter and Elizabeth M. Cohen, Preliminary investigation into the three-dimensional flow through a cascade of aerofoils,** Rep. Memo. aero. Res. Counc. Lond. 2339, 17 pp. (1949).

Cascade data have been obtained for many years as a starting point for design of compressor and turbine stages. Data generally obtained have been taken along midspan of a pack of vanes. Recently it has become evident that such data are inadequate because of effect of boundary layers on tunnel walls at ends of vanes. First analysis of some of these effects is given. Circulation around blades is assumed to trail near side walls whose effects are computed by known method of reflection in walls. In this manner, formulas for difference in flow angles measured at midspan of a finite aspect ratio pack from that for an infinite cascade are computed, and corrected test results are checked against computed two-dimensional flow angles. This is followed by a brief discussion of total drag of vanes in a compressor, results of which could be used in design work. Presented theory leaves much to be developed in future work, since it is not based upon basic equations which control flow in boundary-layer regions, but rather upon approximate estimates guided by experimental results.

Reviewer's Note: Major results are undoubtedly correct since blade circulation must indeed trail as stated by authors. However, experimental results in reviewer's laboratory show that lift on vanes does not fall off appreciably through boundary layer, which is contradictory to assumptions made by authors that lift falls to zero with velocity squares. Howard W. Emmons, USA

2479. **E. P. Neumann and F. Lustwerk, Supersonic diffusers for wind tunnels,** J. appl. Mech. 16: 195–202 (June 1949); discussion, 417–418 (Dec. 1949).

Results are given of experiments on small-scale supersonic constant-geometry diffusers with an initial boundary layer. Experiments are compared with a simple one-dimensional analysis (neglecting friction). Efficiency is defined as work of isentropic compression between initial condition and final pressure, divided by kinetic energy expended.

First, shocks in constant-area tubes were studied by schlieren observations and wall-pressure measurements. The idealized transverse shock was never obtained. For range of Mach numbers covered (1.8 to 4.2), length of shock region (from point of inception to point downstream with maximum pressure) was found to be 8 to 12 tube diameters. Comparison of measured and calculated efficiencies shows good agreement, with a maximum departure of 5%.

Starting and operating conditions are discussed and investigated for five different diffuser types and Mach numbers ranging from 2 to 3. A long parallel throat was found to improve starting and operating efficiencies. A diffuser with a slight divergence of constant area throat gave improved starting characteristics but a small decrease in operating efficiency. Diffusers with rectangular cross section and values of the total angle of convergence of inlet varying from 10 to 50 deg were tested. Best efficiency was obtained with an angle of 10 deg. In the discussion, Kantrowitz compares results of paper with results of his paper on normal shock waves in channel flows (REV 1, 682).

N. H. Johannesen, England

2480. **Flight Research Section Division of Mechanical Engineering Staff, Development of a wing flow apparatus for transonic aerodynamic tests,** Nat. Res. Counc. Canada mech. Engng. Rep. no. MR-6, 8 pp. + 22 fig. (Aug. 1949).

Small models are mounted normal to an aircraft wing and measurements taken in wind-tunnel style of the model characteristics. Wing of the parent aircraft, the plane of symmetry if half models are used, has a modified contour to provide speed gradients in the test section. Range of model Mach numbers is from 0.5 to 1.1 with a range of aircraft Mach numbers from 0.35 to 0.7.

Principal schemes are given for a temperature tube, a free-floating balance, and a five-component balance. With free-floating balance, drag, pitching moment, and rolling moment are sensed by galvanometers driven by strain gage bridges; lift is determined

from pitching moment and knowledge of the geometry of arrangement. The five-component balance measures drag and lift components and pitch, roll, and yaw moment.

Among others, quantitative results are given for temperature measurements versus Ma^2, chordwise, spanwise, and normal Mach number distribution, the latter at model station at three distances and with three arrangements. Some measured wing boundary-layer profiles are also shown to give an indication where to place model end plates.

Disadvantages of method are model spanwise Mach-number gradient and restriction on Reynolds numbers that are inherent in technique. Model wing Reynolds numbers lie in the band 0.25 to 1.1×10^6. S. F. Erdmann, Holland

2481. Julius Lukasiewicz, Effects of air humidity in supersonic wind tunnels, Proc. seventh int. Congr. appl. Mech. 2, part II, 633–640 (1948).

Author briefly reviews results of research on condensation of water vapor in rapid processes (supersonic flows) where it originates in molecular collisions, and gives preliminary tests of a new method of avoiding condensation.

For small and medium-sized supersonic tunnels an adiabatic supercooling of about 50 C is possible. At lower temperatures a condensation shock occurs, changing the mean values of velocity, pressure, etc., as compared with dry flow, and impairing uniformity of the velocity distribution. Pressure ratio across a one-dimensional condensation shock, effect of which may be considered as addition of latent heat, is given by

$$P_2/P_1 = [k/(k+1)](M_1^2 - 1)[1 - (1 - Z)^{1/2}] + 1$$

where $Z = (k^2 - 1)[M_1^2 + 2/(k-1)]M_1^2Q/(M_1^2 - 1)^2$; $k = c_p/c_v = 1.4$ for air; and $Q = q/(c_pT_0)$, i.e., ratio of heat input q to stagnation enthalpy. M_1 is the Mach number upstream of condensation shock. Using this expression it can be shown that variation in Mach number is less than 1% for air with an absolute humidity of 0.0005 or smaller.

Avoiding condensation by initial heating is discussed. Condensation in tunnel nozzle also can be avoided by preliminary forced condensation in an auxiliary nozzle. Several schlieren photographs are given for different ratios ψ = (auxiliary nozzle throat area)/(tunnel nozzle throat area) showing, e.g., that condensation shock is removed from an M = 2.48 tunnel nozzle by use of an auxiliary condensation nozzle with $\psi = 0.782$, ratio of stagnation pressure across which is 0.77. No deposition of drops or ice crystals on model occurred in this case. H. Wijker, Holland

2482. W. F. Hilton, Note on two-dimensional supersonic tunnel interference, Rep. Memo. aero. Res. Counc. Lond. 2332, 6 pp. (1949).

Wave nature of wind-tunnel wall interference in two-dimensional supersonic wind-tunnel experiments is reviewed. Various regions between shock and expansion waves are discussed briefly. A few experimental results illustrate some of the points discussed.
Howard W. Emmons, USA

2483. Th. Zobel, Flow measurement by means of light interference, Nat. adv. Comm. Aero. tech. Memo. 1253, 58 pp. (Aug. 1949); translation of ZWB Forschungsbericht no. 1167 (Feb. 1, 1940).

Memorandum gives a general description of apparatus and of its preliminary application to study of various two-dimensional types of flow (circular cylinder; elliptic cylinder, also with incidence; airfoil section; boundary layers; unstationary fields; two-dimensional analog of a radiator cowl). A few simple formulas are given for calculation of density and pressure from shift of interference fringes, but no particular theoretical developments, either in aerodynamics or optics, are presented. Translation from German has a slight stiffness at certain places.
J. M. Burgers, Holland

2484. O. E. Teichmann, Instrumentation for measuring temperature and pressure of luminous flames of short duration, Heat transf. Fluid mech. Inst., 201–210 (May 1949).

These instruments were used for temperatures up to 3000 K and pressures up to 30,000 psi in chamber of a recoilless rifle. The head of an optical pyrometer, containing two filters, photocells and cathode-follower tubes, was clamped over a quartz window in the rifle wall. A continuous record was obtained of light outputs in broad blue and red bands. Temperatures were calculated by comparing outputs and assuming a black-body distribution.

To check spectral distribution, a high-speed spectrograph was built. This consisted essentially of a grating and a rotating mirror, which passed the spectrum over slit of a photomultiplier 3000 times/sec. The unit was double and covered any two ranges of width up to 2000 Å, between 3800 and 11,000 Å.

Pressure gage used consists of a strain element recording deformation of a ferrule under internal pressure.
H. H. M. Pike, England

2485. Louis Viaud, On the utilization of pressure gages in supersonic flow (in French), Proc. seventh int. Congr. appl. Mech. 2, part II, 443–454 (1948).

Some tests are reported on performance of circular static and total-head tubes in supersonic flows. Author's conclusions are: (1) Position of static holes from nose of tube, to give correct free-stream static pressure, varies with free-stream Mach number. (2) Effect of incidence on measurement of static pressure is small if a ring of more than four holes is used. (3) Conventional static tube gives incorrect readings of stream static pressure when traversed through a normal shock. (4) Assumption of a normal shock at entry to a total head tube is justified. (5) Total head with tubes of different sizes is the same to within 1%.

Author further states that, in some of experiments, total head measured in working section of a tunnel was slightly higher than corresponding measurements in the settling chamber. No explanation could be offered for this phenomenon.
G. M. Lilley, USA

2486. Toshirô Terano, Automatic control of solid temperature, 2nd report (in Japanese), Trans. Soc. mech. Engrs. Japan 15, no. 50, 47–52 (1949).

Automatic temperature control of a solid in which two temperature detectors are placed is discussed. It is assumed: (1) From a heater in a chamber surrounded by this solid, heat quantity $s_1\theta_a + s_2\theta_b$ (θ_a, θ_b are detected temperature variations, and s_1, s_2 are sensitivities of the controller) is supplied; (2) heat capacity of chamber is negligible in comparison with that of solid; and (3) temperatures of in- and out-side surface of solid are equal respectively to those of chamber and surroundings. When temperature of surroundings is varied suddenly, stability characteristics of system are discussed by solving equation for one-dimensional heat flow. Results of theory are confirmed by experiments on a cement block. Humio Tamaki, Japan

2487. Masao Ushida, Temperature control of heat exchangers, (in Japanese), Trans. Soc. mech. Engrs. Japan 15, no. 50, 37–47 (1949).

Automatic temperature control for parallel-flow and counter-flow heat exchangers is treated. A temperature detector is placed in fluid whose temperature is to be controlled, and control

is made by changing rate of flow of another fluid in proportion to detected temperature variation. Under several simplifying assumptions, dynamic characteristics of the system and control error are investigated theoretically. Main results are as follows: (1) When temperature efficiency of heat exchanger is low, to locate detector near exit reduces control error but lowers stability. When efficiency is high, the contrary is true. (2) Generally speaking, parallel-flow type is less stable than counter-flow type. (3) Time lag in detector lowers stability.

Humio Tamaki, Japan

2488. **J. E. P. Dunning, The measurement of the temperature of low density moving gases,** Proc. seventh int. Congr. appl. Mech. 3, 252–265 (1948).

Temperature measurement of low-density flowing gases is studied by use of an aspirated thermocouple over which gas is caused to pass at sonic velocity. At sonic velocity and a recovery factor of 0.87, stagnation (total) temperature is approximately (within $^1/_3$ of a per cent) equal to 1.02 times temperature of an adiabatic thermocouple tip for a range of 1.3 to 1.5 for the specific heat ratio. Thermocouple thermal errors are aggravated by existence of low density gas, but high velocity (sonic) tends to offset this effect somewhat. Commonly known methods must be used for reducing or calculating thermal errors associated with thermocouples in gas streams. Warren M. Rohsenow, USA

2489. **Milton Martin and Paul Golovato, Design of a multifrequency reed-type shock gage with an inertia-operated trigger,** David Taylor Model Basin Rep. 654, 14 pp. (May 1949).

2490. **Samuel Davidson and Emily J. Adams, A theoretical study of the multifrequency reed gage for measuring shock motion,** David Taylor Model Basin Transl. Rep. 613, NS 711-002, 83 pp. (July 1949).

2491. **B. J. W. Grier, Experimental investigation of the type-2 multifrequency reed gage for recording shock motion,** David Taylor Model Basin Rep. 603, NS 711-002, 27 pp. (Sept. 1949).

These three papers describe development of an instrument for measuring transient displacements, velocities, and accelerations in heavy structures such as are found aboard ships. Instrument consists of several reeds, whose natural frequencies vary from 20 to 500 cps, fastened to a column and framework weighing from 20 to 40 lb, which is in turn bolted to the object under investigation. In type-2 model, the paper on which displacement of reeds is recorded by a stylus moves in from 50 to 500 milliseconds a distance up to 2 in., motion being triggered by disturbance itself. Record gives, then, displacement of reeds as a function of time. No external power sources or controls are required. Although theoretical analysis of data is quite complicated, conclusion is reached that this type of gage serves a useful purpose, especially as applied to shock motions in ship structures.

Walker Bleakney, USA

2492. **L. F. G. Simmons and C. F. Cowdrey (Appendix by Geoffrey Taylor), Measurements of the aerodynamic forces acting on porous screens,** Rep. Memo. aero. Res. Counc. Lond. R. & M. 2276, 20 pp. (Aug. 1945; printed 1949).

Measurements are furnished of normal and tangential force coefficients of three wire gauzes representing different degrees of blockage. Other data relating specifically to resistance of screens at right angles are included. Numerous diagrams illustrate results. Analysis is based on a short contribution by G. Taylor which connects normal and tangential resistant coefficients κ_θ and F_θ with pressure drop, velocity of stream, and angles θ under which gauze is placed and under which stream leaves screen.

Georg P. Weinblum, USA

Thermodynamics

(See also Revs. 2389, 2473, 2488)

2493. **C. J. Gorter and J. H. Mellink, On the irreversible process in liquid helium II,** Physica 15, 285–304 (May 1949).

Peculiar properties of helium II, i.e., "fountain effect," convection of heat in slits and capillaries, extinction of temperature waves (second sound) are derived by consideration of a two-fluid model. Viscosity of "normal" fluid is computed and compared favorably with measurements. "Superfluid" is considered to have zero viscosity. Experimental data are reviewed and discussed in terms of the two-fluid model. Myron Tribus, USA

2494. **C. J. Gorter, On the thermodynamics of the two fluid model of helium II,** Physica 15, 523–531 (July 1949).

Thermodynamics underlying the two-fluid model of helium is developed. It is shown that conclusions usually drawn from this model are only valid if special expressions for Gibbs' function of liquid are accepted.

From author's summary by Myron Tribus, USA

2495. **A. Michels, Hub. Wijker, and H. K. Wijker, Isotherms of Argon between 0°C and 150°C and pressures up to 2900 atmospheres,** Physica Hague 15, 627–633 (Aug. 1949).

Pressure-volume-temperature data have been measured for gaseous argon by techniques described previously. Density and pressure-volume product have been tabulated for seven temperatures. Pressure ranges from 0 to 2900 atm, while temperatures are 0, 25, 50, 75, 100, 125, and 150 C. Pressure-volume product has been fitted to a power series in terms of density and various virial coefficients determined. Joseph Kaye, USA

2496. **John A. Goff, Final report of the working subcommittee of the international joint committee on psychometric data,** Trans. Amer. Soc. mech. Engrs. 71, 903–913 (Nov. 1949).

Report sets forth in a logical, precise, and comprehensive manner recommendations of title's subcommittee regarding information on thermodynamic properties of moist air which can claim general acceptance on ground that it is thermodynamically consistant within accuracy of existing knowledge. Formulations employed are based on statistical mechanics rather than on the inadequate law of Dalton.

In logical sequence, recommendations are made for acceptable standard composition of dry air and for general physical constants and conversion factors, followed by formulas for calculation of thermodynamic properties of (a) constituent gases of dry air, (b) standard dry air, (c) water vapor, and (d) moist air.

Formulas for calculation of water-vapor saturation pressures are presented for ice (−100 to 0 C), liquid water (−50 to 100 C) and undercooled liquid (−50 to 0 C).

Report is so condensed and replete with information that it almost represents an abstract of what should be known by everyone who is interested in water and moist air from a scientific and an elevated engineering point of view.

Hans F. Winterkorn, USA

2497. **Serge Gratch, Zero-pressure thermodynamic properties of carbon dioxide,** Trans. Amer. Soc. mech. Engrs. 71, 897–902 (Nov. 1949).

Most reliable values of basic thermodynamic properties of gases and vapors are not those determined by direct experiment, but rather those calculated from spectroscopic data by methods of quantum statistical mechanics. The inherent accuracy of spec-

troscopic measurements is greater, and the properties can be calculated, to a high degree of exactness for condition of zero pressure. These values then serve as a solid foundation for extension to higher pressures. Tables based on spectroscopic data are now available for steam, air, and other gases, and this paper presents a similar table for carbon dioxide at zero pressure. Tabulated properties are: specific heat at constant pressure, enthalpy, reduced entropy, and average specific heat (specific heat divided by temperature). Temperature in degrees Rankine is the argument, the intervals for temperature ranges indicated in parentheses being: 5 deg (100 to 130 deg), 10 deg (130 to 240 deg), 20 deg (240 to 500 deg), 50 deg (500 to 1000 deg), 100 deg (1000 to 2000 deg), 200 deg (2000 to 5000 deg). Values are tabulated to 5, 6, or 7 significant figures, although the physical accuracy is undoubtedly less. It is said that results are in fair agreement with those from calorimetric and acoustic-velocity measurements.

C. W. Smith, USA

2498. C. Kämmerer, On a special form of the equation of state (in German), Ann. Phys. Leipzig 6th suite, 4, 325–330 (Mar. 1949).

A special form of the equation of state of a fluid is proposed. It is

$$p = \frac{RT}{v} - \frac{a}{v^2} + \frac{h}{v^n}$$

where a, h, and n are three constants which are to be determined from the critical state of the fluid. The constants in this equation were calculated for several gases. Equation was then used to calculate vapor pressure as a function of temperature; fair agreement with experimental data was obtained.

Joseph Kaye, USA

2499. G. A. Hawkins and J. M. Smith, Equilibrium constants for seventeen gas reactions, Eng. Bull. Purd. Univers., no. 108, 12 pp. (1949).

Only 13 of these equilibrium constants are independent and about half differ by a factor of 1.5 or more from carefully selected values published by the National Bureau of Standards, over at least part of temperature range. H. H. M. Pike, England

2500. J. Kestin, An approximate method for the calculation of the thermodynamic properties of combustion gases, Proc. seventh int. Congr. appl. Mech. 3, 306–307 (1948).

Part I was published in *Aircraft Engineering* XX (1948), pp. 234–243 & 282. Parts II & III are to be published in same source.

From author's summary

2501. R. Becker, Kinetics of the formation of nuclei and statistical theory of condensation, Paper no. 5473, Disc. Faraday Society no. 5, 55–61 (1949).

Formation of nuclei of condensation in vapor phase is discussed from viewpoint of thermodynamics, kinetics, and statistical mechanics. In thermodynamic treatment, an expression is given for absolute equilibrium concentration of droplets below size of critical nucleus. Constant factor occurring in the term for this concentration is tentatively identified with volume of droplet. At pressures above equilibrium pressure for critical nucleus, kinetic or statistical methods are required. Kinetic treatment developed by Volmer, Becker, and Doering is briefly reviewed. A somewhat more detailed account is given of Mayer's statistical calculation of formation of clusters and of its application to condensation. Relation is obtained between statistical formula for concentration of clusters and thermodynamic formula for concentration of droplets if clusters are interpreted as droplets. Finally, application by Kahn and Uhlenbeck of Mayer's theory to condensation of a perfect Bose-Einstein gas is discussed, and attention is drawn to fact that present theory does not take into account van der Waals forces which are mainly responsible for condensation of helium. Kurt Wohl, USA

2502. R. F. Tangren, C. H. Dodge, and H. S. Seifert, Compressibility effects in two-phase flow, J. appl. Phys. 20, 637–645 (July 1949).

For the two-phase system resulting from a liquid with small gas bubbles inside, equation of isentropic transformation is deduced under simplified assumptions, and turns out to be polytropic. Its application to study of motion in ducts shows, therefore, same general phenomena as for gases. Paper is concluded by some comparison with test. Luigi Crocco, USA

2503. A. E. Rodriguez, A general kinetic theory of liquids. VI. The equation of state, Proc. roy. Soc. Lond. Ser. A 196, 73–92 (1949).

An equation of state based on an approximation for the radial distribution function is studied. This equation contains explicitly the roots of a certain transcendental equation. Main purpose of paper is investigation of the roots of this equation. Original transcendental equation is approximated by an algebraic equation which eventually leads to explicit approximate expressions for the roots. Resulting equation of state has different characteristics from those obtained in usual Mayer theory of condensation. In particular, divergence of cluster series is not found to occur at density of saturated vapor. An attempt (using the Lennard-Jones potential) is made to apply theory to liquid argon.

Courtesy Mathematical Reviews J. M. Luttinger, USA

Heat and Mass Transfer

(See also Revs. 2385, 2423, 2425, 2426, 2446, 2447, 2486, 2487, 2493)

2504. E. F. M. Van Der Held and H. J. Van Den Bold, Conduction of heat in a flame of atomic hydrogen, Physica 15, 305–315 (1949).

Authors try to give a mathematical description of phenomena observed in a vertical flame of atomic hydrogen (in which molecules are dissociated by an electric arc) and described in a paper by Bold and Smit [Physica 12, p. 475 (1946)]. Exact solution of this problem being difficult, authors assume thermal diffusivity and velocity of gas to be constant in order to obtain a useful approximation. However, calculated distribution of temperature (3000 to 3500 K) along axis of flame is in good agreement with measurements carried out by line reversal. Considerable increase of velocity of gas above arc from 300 to 1200 cm/sec can be explained by fact that expanding arc gas collides with jet gas and is driven back by reaction. Z. Horák, Czechoslovakia

2505. W. Traupel, Unsteady heat conduction in plates, cylinders and spheres, Sulzer tech. Rev. no. 3, 12–23 (1949).

Mathematical calculation of nonstationary temperature fields has been solved only in quite simple cases. A more general solution is tried for a problem of larger technical importance, namely, to calculate temperature field at any time and quantity of heat exchanged in a homogeneous solid body of simple geometrical form (flat plate, cylinder, sphere) when this body is surrounded by a heat-transmitting medium, temperature of which is a given function of time t. Coefficient of heat transfer between the surface of solid and ambient medium, density, and thermal con-

ductivity of solid are assumed constant. As a simple initial condition, it may be stated that temperature of solid at time $t = 0$ is same at all points; for all time $t > 0$, body is immersed in medium, temperature of which is given by $\vartheta' = f(t)$.

This general problem now contains a special elementary case for which $\vartheta' = \text{const}$: the heat-transmitting medium is thus in contact with solid at temperature $\vartheta' = 0$ up to the time $t = 0$, when it instantaneously rises to the value $\vartheta' = \vartheta^* = \text{const}$, at which it remains for rest of process. That special case has already been treated by Gröber [Gröber-Erk, Das Grundgesetz der Wärmeübertragung, 2. Aufl. Springer, Berlin (1933)], who found a solution for flat plate, cylinder, and sphere. Author also gives the elementary solution, with diagrams more detailed than those of Gröber-Erk. The step from the elementary to the general solution is done by means of superposition principle: steady variation of temperature ϑ' of medium can be represented by small steps, each unsteady step representing the elementary case; general solution may than be expressed as sum or integral of elementary solutions. Integrations in final equations are generally done graphically, but method allows variations in procedure. As an example, calculation of temperature field occurring in a turbine rotor during heating is carried out; knowledge of temperature differences permits calculation of thermal stresses.

W. Fritz, Germany

2506. **M. Fishenden and A. Kepinsky, Resistance to heat transfer in gap between two parallel surfaces in contact,** Proc. seventh int. Congr. appl. Mech. 3, 193–195 (1948).

Temperature drops along a steel rod of 2 in. diam, sawed in two perpendicular to its axis, were measured under various heat flows. A heat resistance, equivalent to an air layer 0.001 in. or (in a twisted position) 0.002 in. thick, was found. Pressure in the contact was minimal. Convection and radiation losses were accounted for by calculation. O. Maštovský, Czechoslovakia

2507. **N. Rashevsky, A note on the diffusion drag forces,** Bull. math. Biophys. 11, 9–13 (Mar. 1949).

Existence and magnitude of diffusion drag forces is critical to author's theory of cell division. Previously, the estimate of $-RTM^{-1}$ grad c had been given, c being the concentration of the solute. Author now thinks of these forces as of the nature of an aerodynamical force imposed by the motion of the solute, but sustained in part by the solvent. This reduces previous estimates by a factor of the order of 10^{-4}, which brings it into line with some of Landahl's analyses of experimental results [same source, 4, 45–62 (1942)]. A. S. Householder, USA

2508. **N. Rashevsky, Note on a case of nonlinear diffusion,** Bull. math. Biophys. 11, 15–17 (March 1949).

Author constructs a nonlinear diffusion equation having a radially symmetric solution in an infinite uniform medium representing a local accumulation of solute at origin. Without discussing stability of this solution, he infers that suitable nonlinearities might indeed lead to stable accumulations (e.g., in cells) and hence to permanent structures. A. S. Householder, USA

2509. **P. G. Gormley and M. Kennedy, Diffusion from a stream flowing through a cylindrical tube,** Proc. roy. Irish Acad. Sect. A 52, 163–169 (1949).

Let $\psi(r, z)$ denote partial pressure of nuclei that are diffusing through a gas which flows steadily through a tube $r \leqq a$, $z \geqq 0$. Velocity of gas is $V = 2Q(a^2 - r^2)/(\pi a^4)$, where Q is the volume of gas passing through the tube per second. The partial pressure satisfies the equation $D\nabla^2\psi = V\partial\psi/\partial z$, where ∇^2 is Laplacian and D is the coefficient of diffusion of the nuclei. For the approximate solution found here the term $\partial^2\psi/\partial z^2$ is neglected. Under conditions $\psi(a, z) = 0$ and $\psi(r, 0) = \psi_0$, authors derive formulas for the number of nuclei per sec flowing across any section of the tube as a function of the position z of the cross section. They are interested in using such formulas to determine the coefficient D from experimental tests. The Fourier transform with respect to z is employed in solving the partial differential equation in ψ.

R. V. Churchill, USA

2510. **G. A. W. Rutgers, The second radiation constant in Planck's radiation formula,** Physica, 15, 985–989 (Dec. 1949).

Radiant intensity of a black body has been measured in order to obtain new data on second radiation constant of Planck. Measurements are carried out for 3 different wave lengths and measuring methods are briefly described. Results show there is no reason to change internationally adopted value of 1.432 cm deg, despite fact that some recent measurements yielded higher values. E. Haenni, Switzerland

2511. **Fritz Neumann, Simplified calculation of the heat-transfer coefficient of condensers** (in German), Z. Ver. dtsch. Ing. 91, 331–335 (1949).

Calculation of area of heat-transfer surface required for film-wise condensation of vapors is usually based on Nusselt equations for resistance to thermal conduction across condensate film. A trial-and-error computation is usually involved, since resistance of condensate layer depends on temperature drop across it or on rate of condensation.

Neumann presents equations and charts by means of which surface area may be calculated directly without determining temperature of metal surface. Methods may be applied both to vertical tubes and to banks of horizontal tubes. Subcooling of condensate and turbulent flow of condensate are not considered.

R. L. Pigford, USA

2512. **O. P. Bergelin, E. S. Davis, and H. L. Hull, A study of three tube arrangements in unbaffled tubular heat exchangers,** Trans. Amer. Soc. mech. Engrs. 71, 369–374 (May 1949).

Pressure drop and heat-transfer tests on three once-through, vertical tubes, unbaffled units using oil with a medium viscosity are reported. Three tube arrangements, equilateral triangular, in-line square, and staggered square, were used, each with three bulk-oil temperatures and a lower, constant tube-wall temperature. Isothermal and nonisothermal pressure-drop data at three temperature levels are presented. Both staggered tube arrangements show higher heat transfer at given pumping requirements than the in-line arrangement, but difference decreases as turbulent region is approached. Effect of viscosity gradient, which does not allow for free convection or nonuniform flow, is presented for friction during heat transfer.

From authors' summary by Byron E. Short

2513. **O. A. Saunders and S. Smoleniec, Heat regenerators,** Proc. seventh int. Congr. appl. Mech. 3, 91–105 (1948).

Paper gives simplified methods for determining regenerative heat-exchanger sizes and resulting fluid temperatures. Size and arrangement of solid material and passages in exchanger, fluid velocities, length of passages in direction of flow, and reversal times for two fluids are the variables. Simplifying assumptions (negligible in effect) permit use of a relaxation scheme for computations. Computing method utilizes a rectangular network built around thermal capacity of fluids and solid material and heat-transfer rate between fluid and solid. Single sudden change (single blow) and periodic (continual reversal) cases are treated. For the periodic case two parameters, reduced length, and reduced

time periods are used for each fluid. Degree to which exchanger mass is utilized for heat storage and thermal recovery are analyzed for limiting and practical cases. Relation between actual and reduced lengths and times is also analyzed.

Byron E. Short, USA

2514. W. Tipler, An electrical analogue to the heat regenerator, Proc. seventh int. Congr. appl. Mech. 3, 196–210 (1948).

An electrical network is described which permits evaluation of performance of regenerative heat exchangers. It is applicable to the general case where hot and cold periods are of unequal duration. In described form, thermal conductivity of exchanger material must be assumed infinite perpendicular to gas-flow direction. Results presented are restricted to a few illustrative examples. R. Hosmer Norris, USA

2515. C. E. Iliffe, Thermal analysis of the contra-flow regenerative heat exchanger, Proc. Instn. mech. Engrs. 159, no. 44, 363–372 (1948).

Paper provides convenient review of available literature, mostly German, on subject, and adds a new procedure for calculating heat-exchanger effectiveness (thermal ratio) which generally yields better accuracy for a limited amount of labor than the "heat-pole" method of Hausen [ZAMM 11, p. 105 (1931)].

Reasonably comprehensive numerical results are also presented, both for the symmetrical case (hot and cold sides similar) previously treated by Hausen, and for two different sets of unsymmetrical cases, one set for which the "reduced-surface ratio" between hot side and cold side is 2.0, and the other for which it is 3.0.

Although results are derived on customary assumption of infinite conductivity perpendicular to gas-flow direction, various previous attempts to evaluate the error from this assumption are reviewed in considerable detail. R. Hosmer Norris, USA

Acoustics

2516. John W. Miles, On the change of cross section and bifurcation of a cylindrical tube, J. acoust. Soc. Amer. 22, 59–60 (Jan. 1950).

Note is concerned with propagation of plane sound waves through two types of cylindrical tubes, one having an abrupt decrease of cross-sectional dimension, and the other an abrupt division into two branches, affected by an enclosed concentric cylindrical sheath. Without details of analysis, some numerical comparison of results for the two problems is given, and an earlier conjecture of their equivalence is disproved. H. Levine, USA

2517. W. Chester, The propagation of sound waves in an open-ended channel, Philos. Mag. 7, 41, 11-33 (Jan. 1950).

In 1948 Levine and Schwinger studied radiation and transmission properties of a semi-infinite circular acoustical duct under certain conditions of excitation [Phys. Rev. (2) **73,** 383–406 (1948); Rev 3, **986**]. It was found that problem could be formulated as an integral equation of Wiener-Hopf type and could be solved for this case. From the solution pertinent physical parameters could be found, i.e., radiation pattern and reflection coefficient. Inspired by this work, author does the problem of a semi-infinite pair of parallel plates, excited by the dominant acoustical mode. It is found that problem may still be formulated as an integral equation of Wiener-Hopf type and all pertinent physical parameters obtained. [Reviewer's comments: This acoustical problem is equivalent to a problem in electromagnetic excitation of so-called *e*-plane polarization and has been done by L. A. Vainstein [Izvestiya, Ser. Fiz. 12, 144–165 (1948)]. The *h*-plane polarization has been done by A. E. Heins [Quart. appl. Math. 6, 157–166, 215–220 (1948)] and L. A. Vainstein (op. cit). A. E. Heins, USA

2518. W. J. Cunningham, Application of vector analysis to the wave equation, J. acoust. Soc. Amer. 22, p. 61 (Jan. 1950).

Content of this letter to Editor has been previously treated by Heymann, "Die Differentialgleichungen des Schallfeldes," Akus. Zeits. 2, p. 193 (1937). Vincent Salmon, USA

2519. Harold Levine, Variational principles in acoustic diffraction theory, J. acoust. Soc. Amer. 22, 48–56 (1950).

Diffraction of a plane harmonic sound wave by an aperture in an infinitely thin rigid plane screen is investigated theoretically. Variational principles for diffracted spherical wave amplitude at large distances from aperture are derived. In first of these, stationary property is exhibited for the class of functions comprising normal derivative of aperture velocity potentials, whose distributions are governed by a generally insolvable integral equation. Second involves functions which characterize discontinuity in velocity potentials at screen (or their deviation from infinite screen distributions), and are specified by another integral equation. A comparison of the two variational principles is given, which indicates that their over-all agreement following use of approximate functions is a measure of accuracy obtained.

Plane wave transmission cross section of aperture is related to the imaginary part of diffracted amplitude in direction of incidence and is cast in stationary forms. Particular attention is given to low and high frequency behavior of various forms of cross section, including comparison with Kirchhoff theory predictions. (From author's summary.) Reviewer notes treatment is compact and refers frequently to author's earlier papers [H. Levine and J. Schwinger, Phy. Rev. I, 74, p. 958 (1948); II, 75, p. 1423 (1949)]. John W. Miles USA

2520. M. Mokhtar and M. Shehata, Scattering of ultrasonic waves in gases, J. acoust. Soc. Amer. 22, 16–19 (Jan. 1950).

Diffraction by a rectangular slit of ultrasonic waves in air, O_2 and CO_2 is studied using a hot-wire anemometer. At frequencies well below 1000 kc/sec, diffraction laws are obeyed in air and O_2 but a diffuse pattern is obtained in CO_2. Authors conclude that in anomalous-dispersion region a gas scatters sound. At 1000 kc/sec, scattering is observed in air and O_2 as well, suggesting dispersion in these cases. It is not clear to reviewer how this is to be reconciled with lack of an associated increase in velocity.

Martin Greenspan, USA

2521. Robert M. Hoover, Donald T. Laird, and Laymon N. Miller, Acoustic filter for water-filled pipes, J. acoust. Soc. Amer. 22, 38–41 (Jan. 1950).

Acoustic noise with components past 140 kcps occurred in auxiliary equipment for a water tunnel, with noise level transmitted by water path exceeding that through pipe path by about 30 db. Low-frequency attenuation of as much as 45 db in water path was secured by inserting a perforated panel of pressure release material in series with water stream to form a high-pass filter for radial nodes. Material consisted of many small noninterconnecting air cells in a matrix of suitable elastomer; thus walls of perforations provided a radial stiffness that was low compared to water. Pressure release afforded by this panel accounts for considerable attenuation of radial nodes by which much of high-frequency noise was transmitted. Pipe-transmitted noise was reduced by a multiple sandwich of soft elastomer gaskets bracketing panel.

Reviewer believes that results and their presentation would have been improved by more attention to analytical considerations, especially as regards what modes are transmitted and measured. However, empirical results were apparently sufficiently satisfactory not to warrant this step.
Vincent Salmon, USA

2522. **Francis E. Fox and Virginia Griffing, Experimental investigation of ultrasonic intensity gain in water due to concave reflectors,** J. acoust. Soc. Amer. 21, 352–359 (July 1949).

Production of high-intensity sound waves in liquids by concave reflectors is reported. By using a beam of ultrasonic waves of about two watts of acoustic power and focusing it with a watchglass, an ultrasonic fountain 10-cm high was produced. Distribution of intensity in focal region was determined by measuring screening effects of properly placed obstacles. Intensity was determined from deflection of suspended beads. Good agreement between theory and experience was found. No cavitation was observed even though negative pressures of 41 atm were produced.
Gerald Pickett, USA

2523. **G. W. Willard, Focusing ultrasonic radiators,** J. acoust. Soc. Amer. 21, 360–375 (1949).

High-intensity ultrasonic waves were produced in liquids by use of spherical-shell piezoelectric radiators. Waves came to a focus at center of curvature of shell. Sharpness of focus and radiation efficiency of different areas of radiator were observed by means of ultrasonic light-diffraction pictures. Theoretical calculations are made to show that highly efficient spherical radiators could be made if proper variation in thickness of shell is made to compensate for variation of elastic and piezoelectric constants with orientation of radiating area. Gerald Pickett, USA

2524. **M. Gosewinkel, Measuring loudness** (in German), Arch. techn. Messen, no. 155, 70–71 (June 1948).

A survey of subjective and objective methods used in Germany. Contains nothing new. R. Vermeulen, Holland

2525. **Osman K. Mawardi, Generalized solutions of Webster's horn theory,** J. acoust. Soc. Amer. 21, 323–330, (1949).

Paper discusses methods of solving Webster's equation for approximate treatment of (assumed) plane wave propagation in horns of variable cross section [Proc. nat. Acad. Sci. 5, p. 275 (1919)]. Electrical analog of a transmission line with varying parameters is used as an aid in reducing problem of infinite horn to the Ricatti equation $\partial Z/\partial x - j\omega SZ^2/\rho c^2 + j\omega\rho/S = 0$, where Z is local impedance and S horn area. Application of electrical analog to finite horns is also discussed, and author promises further development of this case in a separate paper. In second part of paper, reduction of Webster equation to the canonical form $\phi_{ss} + (A_0 + A_1/x)\phi_s + (B_0 + B_1/z + B_2/z^2)\phi = 0$ is discussed. The physical requirement that horn-generating function be independent of frequency restricts one of the constants, but variation of remaining four gives rise to a fourfold infinity of families of horns. Examples are given John W. Miles, USA

2526. **Alfred Leitner, Notes on diffraction by a circular disk,** New York University, Washington Square College, Math. Res. Group, Res. Rep. No. EM-12, i +36 pp (1949).

Report investigates diffraction of a plane sound wave from a rigid circular disk of zero thickness. Recent exact calculations of C. J. Bouwkamp [Diss. Gronirgen (1941)] and R. D. Spence [J. of Am. Per. 20, p. 380 (1948); REV 2, **527**] are extended in order to compute and discuss values for diffracted fields near disk and far from it. Kirchoff approximation is compared to exact solution and is found to give more favorable results for the class of configurations treated here. Reason for this is discussed in some detail. A. E. Heins, USA

2527. **Francis M. Wiener, The diffraction of sound by rigid disks and rigid square plates,** J. acoust. Soc. Amer. 21, 334–347 (July 1949).

Previous investigators have used an approximate method of solving for pressure at center of plane surface of an obstacle when exposed to plane progressive waves. Author extends method to solve for pressure distribution over whole surface of rigid disks and square plates. Method consists of treating scattered portion of total pressure as though it resulted from an assumed virtual vibration of plane surface. This is a mathematical device for satisfying the boundary condition of particle velocity at surface of plane obstacle.

Computed results are plotted and compared with rather extensive experimental results obtained by author. A few results obtained by more exact theory using solutions of wave equation in oblate spheroidal coordinates are also shown. The approximate theory was found to be in good agreement with experiment for the "illuminated" side, and for an incident angle of 180 deg on "shadow" side. For other incident angles on shadow side, agreement was poor.

Chief advantage of approximate theory is its relative simplicity compared with exact solution of wave equation.
Gerald Pickett, USA

Ballistics, Detonics (Explosions)

(See also Revs. 2429, 2484)

2528. **Raymund Sänger, Influence of a gust on projectile trajectory** (in German), Helv. Phys. Acta 23, no. 1/2, 143–158 (1950).

Let sudden squalls of wind be superimposed upon a general uniform motion of the air. Velocity of squalls is assumed to be independent of altitude z. Under this assumption author confines himself to consideration of dispersion of points of impact on horizontal xy-plane through initial point of trajectory. The mathematical expressions of the two horizontal and mutually perpendicular velocity components $w_x(x, t)$ and $w_y(y, t)$ of the superimposed wave motion, called squally range wind and squally cross wind, are both assumed to be of the form

$$w_x(x, t) = w_0 + A \cos\left[\frac{2\pi}{T}\left(t - \frac{x}{u}\right) + \delta\right]; \quad u = \frac{g}{2\pi} T$$

and similarly for $w_y(y, t)$. According to experiment, the period T varies between 3 and 60 sec and the velocity amplitude A can assume values up to 10 m/sec. Results of paper may be summarized as follows. While range effect of a squally range wind may remain relatively small even if period T and amplitude A of wind both assume their maximum values (in most cases range effect is almost negligible), a deflection effect due to squally cross wind may possibly constitute main part of actually observed deflection. E. Leimanis, Canada

2529. **C. G. Maple and J. L. Synge, Aerodynamic symmetry of projectiles,** Quart. appl. Math. 6, 345–365 (1949).

Validity of "aerodynamic hypothesis" (air reactions on a rigid body depend only on instantaneous linear and angular velocities u and ω) is assumed, and resultant force and moment components are expanded in power series in four transverse variables $u_1 \pm iu_2$, $\omega_1 \pm i\omega_2$, coefficients of these power series depending on the two longitudinal variables u_3 and ω_3. The 3-axis is assumed (a) to be

an axis of n-gonal rotational symmetry, (b) to be (together with 1-axis) in a plane of reflectional symmetry of projectile. Restrictions that assumed symmetries imply for coefficients are investigated.

It turns out that in case (a) certain coefficients must always vanish; for instance, if n is even, transverse components can only have terms of odd degree and longitudinal components only such of even degree. A complete survey of "surviving" terms is given for $n = 2, 3, \ldots, 8$. In case (b) (which is known from longitudinal airplane stability) only conclusions about the parity of the coefficients regarding variable ω_1 are possible.

Further-going restrictions can be obtained when (a) and (b) occur simultaneously and projectile is constrained to have no axial spin or no spin at all. G. Kuerti, USA

Soil Mechanics, Seepage

(See also Revs. 2283, 2287, 2368, 2395)

2530. L. Casagrande, Electroosmosis in soils, Géotechnique Lond. I, 159–177 (1949).

Paper shows that, under a constant potential gradient between cathode and anode, the electroosmotic transport of water may be assumed to be constant for all soils. Osmotic permeability was found to be 5×10^{-5} cm/sec for 1 volt per cm for 12 various soils. Discussion covers a large amount of experimental investigation on the amount of flow under potential gradients from 0.1 to 12 volts/cm, on water distribution between electrodes, on stresses developed by electroosmosis, and on necessary consumption of electrical energy. Field of practical application is likely to be limited by economic factors. Diverting flow of pore water from cuttings to stop boiling at bottom of excavations is a possibility. Electroosmosis also has a favorable effect as an indirect source of consolidation of soft-soil deposits. Electroosmotic forces can achieve transmission of stresses in pore water to particles of soil, and so mobilize the full weight of soil plus water. Therefore, it will be sufficient to introduce electrodes locally, and to use relatively small amounts of energy. Author sees in this an interesting possibility of leveling back buildings which have settled nonuniformly.

Some information is also given about the electric equipment and layout necessary for use of electroosmosis in foundation engineering. R. Spronck, Belgium

2531. Friedrich Schiel, Bearing capacity of foundations of arbitrary cross section (in German), Abh. Bodenmech. Grundbau, pp. 47–50, Berlin, Erich Schmidt Verlag (1948).

An approximate procedure is proposed for computation of bearing capacity of foundations on granular soils, based on assumption that foundation pressure increases linearly from edges toward center of bearing surface. Author shows that his formulas reproduce results of bearing tests with footings of various shapes fairly well. Extension of procedure to eccentric loading is given without experimental corroboration of results.

A. M. Freudenthal, USA

2532. Friedrich Schiel, Determination of the grain form (in German), Abh. Bodenmech. Grundbau, pp. 63–65; Berlin, Erich Schmidt Verlag (1948).

The author presents two methods for the determination of grain shape which might be well applicable in soil mechanics. The first might be called "shape-sieving" and is based upon fact that with perforated sieves only diameter of particle exerts an influence of its passage, while with wire-mesh sieves it is a question of width and thickness as well.

The ratio u of the volumes passing through a perforated sieve l and through a wire-mesh sieve m of same dimensions expresses the sphericity K of the material: $K = 100(1-\log u/\log 2^{1/3})$.

The second way of determining grain shape is based on permeability of material, which is dependent upon pore-volume n and on specific surface area S. With reference to Carman's experiments, a shape indicator is developed: $\psi = 20.7 S/_0\!\int^{100} dy/V^{1/3}$ where V denotes the average grain volume in a small dy range of the sieve-curve; $S = 122[n/(n-1)](n/k)^{1/2}$; and k is the permeability coefficient. Ch. Szécchy, Hungary

2533. Edgar Schultze, Composition and resolution of slip lines (in German), Abh. Bodenmech. Grundbau, pp. 34–45, Berlin, Erich Schmidt Verlag (1948).

In earth pressure on walls, the old wedge theory considered only failure line bounding the wedge, which, in a first approximation, was assumed to be a straight line. Many attempts have been made to find exact form of this line, and several authors give its differential equation, but its integration raises great difficulties because of boundary conditions which are connected with movement of wall. As these movements cause changes in whole mass of backfill and produce various slip lines within mass, an exact mathematical solution cannot be given. Author, using kinematic considerations, gives an approximate method for determination of these slip lines in several cases, and compares results with experiments on models. The two show sufficient concordance for practical problems. Aurel A. Beleş, Rumania

2534. Jacques Verdeyen, The use of flat sheet-piling in cellular construction, Proc. sec. int. Conf. Soil Mech. Found. Engng 7, 61–64 (1948).

Author describes several practical applications to construction of cofferdams, and presents adopted methods of calculation of stability, width, interlocking joints, driving depth, etc. Special attention is called to necessity of drainage and, if the cofferdam is exposed in open sea, to danger of wave action.

Ch. Széchy, Hungary

2535. Waterways Experiment Station, Flood-control project for Johnstown, Pennsylvania, Wways. exp. Sta. tech. Memo 2-303, 88 pp. + 129 plates (Nov. 1949).

Model study, conducted by WES in Vicksburg, Miss., was made to determine most effective and economical plan for improvement of channels of Conemaugh River at Johnstown, Pa., so that floods as serious as can be expected would be carried out without excessive damages. The experimental study, on a model with different linear scale ratios (geometric distortion of 2.5), is very important; documentation is copious and careful. Model technique is interesting in the correction applied to discharge scale ratio to overcome difficulty of reproduction of prototype roughness. Duilio Citrini, Italy

2536. A. W. Skempton, The effective stresses in saturated clays strained at constant volume, Proc. seventh int. Congr. appl. Mech. I, 378–392 (1948).

Author starts with concept that effective stresses acting on a soil mass may be expressed as total applied stresses minus pore water pressure. From this he develops expressions for effective stresses in terms of applied stress increments (deviator stress) and compressibility characteristics of clay. Limiting values are placed on compressibility ratio, depending on state of consolidation of a clay and its sensitivity to structural breakdown under load.

Developed concepts were checked for Wiener Tegel clay using results of previous tests performed by Hvorslev and Rendulic. Reasonably good agreement was obtained between computed pore

water pressures and effective stresses and those observed in tests. A similar qualitative comparison was made for Boston blue clay.

Method was further developed to obtain expressions for compressive strength in triaxial shear test and in plane strain, the latter a case often arising in engineering problems. It is demonstrated that the expression gives same result as that obtained by conventional approach where angle of internal friction is assumed zero. That the angle of friction is not actually zero has been observed in actual measurements of failure planes in field and laboratory; proposed concept satisfactorily explains this apparent paradox. Woodland G. Shockley, USA

2537. **D. W. Taylor, Shearing strength determinations by undrained cylindrical compression tests with pore pressure measurements,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 5, 45–49 (1948).

Results of triaxial shear tests on two normally loaded clay samples are shown. Applied axial and lateral pressures, as well as resulting pore pressures, were measured during shearing of samples; intergranular principal stresses were thus determined. A plot of ratio of shearing stress S to major principal stress σ versus axial strain for the two clays, reveals a close similarity between them even though properties are very different. Author suggests that upper and lower limits of the S/σ_1 can be used to estimate maximum and minimum values of S for a clay consolidated under principal stresses whose ratio is greater than those corresponding to smaller strain for the S limits. He further suggests that since these limiting values are usually not far apart, the lower one would be a conservative value to use for all overburden pressures σ_1, and all ratios of principal stresses. For precompressed clays, these upper and lower limits can serve only as a check on shear values determined by more direct methods.
Eben Vey, USA

2538. **H. R. Cedergren, Use of flow-net in earth dam and levee design,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 5, 293–298 (1948).

An interesting and useful discussion of use of flow nets with particular reference to effects of stratification and the case of a section composed of zones with different permeabilities. Eighteen carefully drawn flow nets are given for various dams and levees, the most striking being effect of stratification in a lower layer in ground beneath a levee. If this layer is assumed to be homogeneous, exit gradient in upper foundation layer, at the toe of levee, is 0.7. But if lower layer is stratified, with a ratio of horizontal to vertical permeability of 9 (a very moderate degree of stratification), exit gradient is 1.9. This shows both importance of obtaining an estimate of the ratio of horizontal to vertical permeabilities in any stratum before carrying out a flow net construction, and great limitation of accuracy in seepage problems. Author rightly emphasizes that no computation is more dependable than the simplifying assumptions upon which it is based, and this applies particularly to flow through earth dams and levees. Elaborate mathematical analyses are a waste of effort in these problems. A. W. Skempton, England

2539. **R. J. Hank and L. E. McCarty, Shear failure in anisotropic materials possessing any values of cohesion and angle of internal friction,** Proc. Highw. Res. Bd. Ann. Meet. 28, 449–455 (1948).

This is an attempt to predict the strength of anisotropic soils subjected to general triaxial states of stress. The anisotropy of the soil is defined by two orthogonal planes of minimum and maximum shear strength; furthermore, the validity of the Coulomb-Mohr strength hypothesis is assumed.

No acceptable analytical proof or experimental evidence is offered to support the method promulgated; it is shown, however, that method furnishes results that are consistent with those obtained by A. Casagrande and N. Carillo [J. Boston Soc. civ. Engrs. (April 1944)] in the two limiting cases of purely cohesive material and cohesionless material. Oscar Hoffman, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 2384, 2391, 2446)

2540. **W. L. Godson, A study of the deviations of wind speeds and directions from geostrophic values,** Quart. J. roy. met. Soc. 72, 3–15 (Jan. 1950).

Paper is one of a number of efforts by several investigators to determine absolute magnitude of deviation of wind velocity in free atmosphere from geostrophic velocity (velocity under conditions of horizontal, unaccelerated, inviscid flow). By assuming inviscid flow, geostrophic velocity and angle between actual wind and geostrophic wind are expressed by two simultaneous equations involving velocity components, which are computed from wind velocities observed at 6-hr intervals during a 3-day period. Space and time derivatives are determined by subjective analysis. Mean absolute deviation of wind direction from geostrophic direction is about 15 deg and mean absolute deviation of speed is about 30%. Terms in equations involving vertical component of velocity are found to be of same order of magnitude as terms involving horizontal component of velocity. These results are consistent with results of other studies using other methods and different weather situations.

Reliability of quantitative results obtained by author depends critically upon density and accuracy of observed wind velocities and computational methods used in determining simultaneous values of velocity components. Reviewer believes that importance of these points warrants considerably more attention than is paid to them. As a single important example of a source of error which may invalidate the results: vertical component of velocity is determined at a single time by a method (vertical summation of estimated horizontal velocity divergence at a point) which is recognized generally as being subject to large error. Further, most investigators agree that vertical component of velocity at a single place and time frequently undergoes large changes during a period of 6 hr or less. Therefore, author's use of the "instantaneous" values of vertical velocity in equations involving time derivatives obtained from intervals of 6 hr is likely to introduce errors which are difficult to evaluate in individual cases. For this reason, another method of computation used as a check would have been of great help in estimating reliability of quantitative results. Robert G. Fleagle, USA

2541. **Ronald L. Ives, Remote indicating and recording wind equipment, I and II,** Bull. Amer. met. Soc. 30: 321–325, 352–356 (Nov. and Dec. 1949).

2542. **Warren L. Godson, Some remarks concerning the gradient-wind equation,** Bull. Amer. met. Soc. 30, 342–346 (Dec. 1949).

In order to point out assumptions contained in the equation, a short derivation is given of gradient-wind equation involving motion of pressure systems. A result is obtained from gradient-wind equation, similar to that deduced previously for closed highs and lows aloft, namely, that slowly moving cold lows and warm highs should move in direction of strongest winds with a speed of translation of the order of the difference between maximum and minimum circumferential winds. A discussion, based

on this equation, is also given for speed of movement of pressure ridges.

Solutions of the equation for anticyclonic contour curvature are discussed. It is concluded that the classically discarded root of the equation is a valid root and that its use leads to an explanation of the so-called super-gradient winds. It is also stated that for previously discarded root either accelerations or decelerations act to further increase rather than to decrease wind speed.

Neal Tetervin, USA

2543. Max Hinzpeter, Measurements of air pressure with Graw's radiosonde and the correction equation for instrument calibration (in German), Met. Rdsch. 1, 483–488 (1948).

Systematic errors inherent in Graw's radiosonde are investigated to determine whether pressure measurements indicated by instrument after elimination of known errors are within allowable tolerance of ±2.5 mb.

Graw's radiosonde measures barometric pressures at various altitudes by means of a double aneroid. Relative extension of bellows is changed to rotary motion of an arbor carrying ten arms by means of a suitable linkage. Contactors at extremities of these arms engage a rotating segment driven by a constant speed motor so that, when contact is made by a certain arm, an electric circuit is closed which in turn gives rise to a radio signal sent out into space. The signals vary for the different arms so that position of arbor, and thus pressure at any instant, can be identified from signal received at the ground station.

Pressure measurements received from instrument are subject to two errors; one arises from effect of temperature on the multiplying linkage, and the second from effect of temperature on gas contained in pressure cells. These temperature errors were investigated and presented in form of an equation which lends itself to graphical presentation for convenient use.

From detailed investigation of the correction equation, it is concluded that instrument is least sensitive to temperature differences if compensating pressure in pressure cells is equal to 870 mb for steel diaphragm cells, and 596 mb for teryllium diaphragm cells. It is further concluded that the allowable tolerance of ± 2.5 mb in pressure measurements can be obtained after making corrections for temperature effect. Karl E. Schoenherr, USA

2544. Alf Nyberg, An aerological study of large-scale atmospheric disturbances, Tellus 1, 44–53 (Feb. 1949).

Pressure change areas in the 500-mb level move with 60% of speed of wind in that level. Advection of cold (warm) air at upper isobaric levels is followed by geopotential falls (rises) at the same levels. Prognostic usefulness of this rule is much higher in 500-mb level than in 700-mb level. There are strong indications of dynamic interaction between flow patterns in distant regions on hemisphere. A deepening of one upper trough will, in special cases, lead to a second deepening downstream. In a deepening trough the cold tropospheric air is shown to be sinking at the same time as the jet stream increases and the front becomes sharper. Models of the distribution of vertical motion and divergence in the frontal zones are given.

From author's summary by Hans F. Winterkorn

2545. F. H. Schmidt, On the theory of small disturbances in equatorial regions, J. Met. 6, 427–428 (Dec. 1949).

Irregularity of weather in an equatorial region such as Indonesia is ascribed to small disturbances, many of which move as an east-west lying "through" toward the equator. Java is visited, from time to time, by such small disturbances which originate from the southern Indian Ocean, move northward, and disappear gradually on approaching the equator. Application of method of small disturbances is made difficult by the significant variation of the coefficients in the equation. This is demonstrated by mathematical formulation given of problem, which serves as a good illustration of reasons why equatorial meteorology offers theoretically, as well as practically, such great difficulties. Hans F. Winterkorn, USA

2546. Ernst Holder, The differential equation for the stream function of the steady rotationally symmetrical circulation of an adiabatically varying frictionless atmosphere over the earth (in German), Met. Rdsch. 1, 449–450 (Sept.-Oct. 1948).

The differential equation for the (Stokes) stream function for frictionless, adiabatic, nonisentropic flow is derived, assuming axial symmetry and a gravity potential. J. S. Isenberg, USA

2547. Koji Hidaka and Shigeo Hikosaka, Note on the Hawaiian tsunami of April 1, 1946 (in Japanese), J. ocean. Soc. Japan 5, 19–31 (1949).

Investigation is an application of J. Proudman's theory of diffraction of tidal waves by a circular island to distribution of tsunami heights along shore of Kauai island, Hawaii. Calculated heights agreed approximately with theory.

S. Syōno, Japan

2548. J. Valembois, The determination of wave characteristics by measurements of deep pressure variations (in French), Houille blanche, no. spec. B, 707–715 (1949).

Author discusses merits and obstacles of various types of recording manometers that can be used on ocean floor unattended for a month. A differential manometer has been developed by author and some field tests were carried out in estuary of Rance. Instrument consists of two chambers with a small pressure leakage between them. Time constant of system is chosen to be sufficiently large so only the relatively faster pressure variations are recorded. Sensitive element is a strain gage, and recorder is located on shore connected by underwater cable.

A long discussion with many participants completes publication. Leslie S. G. Kovasznay, USA

2549. Takashi Ichie, On the wave caused by an oscillating pressure (in Japanese), J. Oceanogr. Soc. Japan, pp. 1–18 (Aug. 1949).

In order to explain, theoretically, swells caused by pressure variation of short period in a typhoon center, author solves a Cauchy-Poisson wave problem for case where depth of sea and dimension of source are both finite. Five types of pressure distribution are treated. Relations between wave length, amplitude, and dimension of source are discussed and results are shown graphically. S. Syōno, Japan

2550. Shigeo Hikosaka, On the diffraction of tidal waves by an island with irregular boundary (in Japanese), J. ocean. Soc. Japan 5, 32–37 (Aug. 1949).

Author made an attempt to solve the problem of diffraction of tidal waves by an island with an irregular boundary by means of I. Tuboi's method for free oscillation of water. Selecting properly (25 + 1) different points on boundary, he determined general solution so as to satisfy boundary condition at these points.

S. Syōno, Japan

2551. Koji Hidaka, Mass transport in ocean currents and lateral mixing, J. Mar. Res. 8, no. 2, 132–136 (1949).

A mathematical technique which has had wide use in the theory of elasticity and other boundary-value problem of mathematical physics, the calculus of variations, is employed by author for

establishing a formal solution to the important problem of determining mass transport of horizontal currents of an enclosed sea, in presence of frictional forces due to lateral diffusion of momentum by turbulence. Unlike the approach of Sverdrup [Proc. Nat. acad. Sci. 33, no. 11, 318–326 (1947)] and Munk [J. Meteor. 7, no. 2, 79–93 (1950)], author considers pressure (or distribution of mass) as one of the prescribed variables, in addition to wind stress. As a consequence, solution for mass transport depends essentially upon wind stress and pressure gradient, rather than on vorticity of wind stress only, as in the papers by Munk and Sverdrup. Theory yields no explanation for the field of mass as maintained by wind stress, but does point way toward a more rational means of computing transport from hydrographic observations, other than by use of simple geostrophic formula (or Bjerknes theorem). No computations are given in present paper, but are presented in a later work by author [Geophysical Notes, Tokyo Univ. 3, no. 2, 1–11 (1950)].

Robert O. Reid, USA

2552. **B. Gutenberg, On the layer of relatively low wave velocity at a depth of about 80 kilometers,** Bull. seism. Soc. Amer. 38, 121–147 (Apr. 1948).

Earlier work of author on interpretation of seismograms at epicentral distances between 4 and 26 deg is elaborated and developed. Revised P and S travel-time curves are given which consist of two branches: the first is almost a straight line ending in a shadow zone; the second is delayed relatively to first at beginning where it is associated with relatively large amplitudes. Various trial velocity distributions are assumed and observations are found to be compatible with assumption that there is a decrease of $^1/_2$ to 3% in both P and S velocities with increase of depth, leading to minimum velocities at 80 to 100 km below the Mohorovičić discontinuity. No decision is given on rapidity of setting in of these velocity decreases, but it is concluded that a layer of relatively low velocities extends down to a depth of 150 to 180 km, velocities increasing rather rapidly below this last depth. Bearing of results on "20 deg discontinuity" is discussed.

K. E. Bullen, Australia

2553. **Woodrow C. Jacobs, The energy acquired by the atmosphere over the oceans through condensation and through heating from the sea surface,** J. Met. 6, 266–272 (Aug. 1949).

Author presents results of his calculations of net rate of transport of energy from ocean to atmosphere by conduction, evaporation, and precipitation. Results are presented in form of Northern Hemisphere maps having isopleths of constant rate of energy transport (cal cm^{-2} day^{-1}) for winter and summer season. Only final results are presented; however, date sources and methods of correction are discussed. Final results are in reasonable agreement with similar calculations made from independent data by Wexler (1944).

H. J. Stewart, USA

2554. **J. L. Orr, D. Fraser, and K. G. Pettit, Analysis of experiments on inducing precipitation,** Nat. Res. Counc. Canada Mech. Engng. Rep. no. MD-32, 10 pp. (Aug. 1949).

Fifty-nine clouds were seeded using dry-ice technique. Half the tests were made on selected supercooled cumulus clouds from which precipitation reached ground on 43% of trials. There was a marked enhancement of chances for ground precipitation when cloud depth exceeded 0.75 of cloud base height. Some experiments in cloud forming by seeding supersaturated air are given. The 59 Canadian tests are compared to 20 Australian tests of similar nature to show general agreement in results. Authors conclude that they were instrumental in causing precipitation in a large number of cases, and that dry-ice seeding technique should be useful for forest-fire prevention, not by directing rainfall to put out a fire, but by increasing precipitation and humidity near the ground during dry seasons.

Myron Tribus, USA

Lubrication; Bearings; Wear

2555. **E. B. Greenhill, The lubrication of metal surfaces by mono- and multi-molecular layers,** Trans. Faraday Soc. 45, 631–635 (July 1949).

Work is an extension, under more carefully controlled conditions, of previous work by other authors, whose papers are mentioned. It concerns the assessment of the Bowden and Leben "stick-slip" apparatus, of lubricating properties of mono- and multi-molecular layers of stearic acid, three stearates, and octadecyl alcohol alternatively, which were deposited from a Langmuir trough on one only of the two mating rubbing surfaces of similar metals, whereas the other surface was *not* lubricated beforehand. Lubricating properties were judged by two criteria: occurrence or nonoccurrence of "stick-slip," presumably at ambient temperature only, and level of transition temperature (up to about 150 C) at which lubrication failed, as evidenced by a marked increase in friction. Reviewer would point out that results obtained on apparatus used are greatly affected by its vibrational characteristics, so that caution is needed if it is desired to extend the results to conditions other than those obtained here.

It was found that, in terms of the minimum number of molecular layers required for preventing stick-slip, metallic stearates, either applied as such or formed *in situ* by reaction between stearic acid and more reactive metals, are more effective than stearic acid, if that remains unchanged, as would occur with less reactive metals; steel is most easily lubricated. Nonreactive silver is found to be lubricated efficiently, if as few as three molecular layers of silver stearate are applied as such.

Also, from standpoint of transition temperature, support is found for view suggested by Bowden and associates [Nature 156, p. 97 (1945)] that with reactive metals, such as steel and copper, it is properties of soap film formed *in situ* from fatty acid applied which are decisive, and not those of fatty acid.

H. Blok, Holland

2556. **A. Cameron, The surface-roughness of bearing-surfaces and its relation to oil film thickness at breakdown,** Proc. Inst. mech. Engrs. 161, no. 48, 73–79 (1949).

Using a tilting-pad bearing apparatus, oil-film thickness at first contact and at seizure was studied for steel surfaces having various surface roughnesses run against white metal. First contact was detected by change in electrical resistance and seizure by rise in friction torque. Water, soap solutions, paraffin, and light oil were used as lubricants. It is found that initial contact occurred when the outlet film thickness, calculated from normal hydrodynamic theory, fell to three times the *maximum* surface roughness and seizure occurred when it was double the *average* roughness. Little difference is found among the four lubricants. With fine finishes the surfaces sometimes seized on initial contact, suggesting a drawback to having too smooth a finish.

John T. Burwell, USA

2557. **T. E. Norman and C. M. Loeb, Jr., Wear tests on grinding balls,** Trans. Amer. Inst. min. metall. Engrs. 176, 490–526 (1948).

Authors propose to ascertain which composition and heat treatment give steel balls a higher wear resistance. Some of conclusions are: (a) Wear is proportional to ball-surface area; (b) martensitic steel balls, with retained austenite, are most wear-

resistant, provided austenite is capable of turning into martensite under operating conditions; prior austenitic grain size does not influence wear resistance; (c) carbon content improves wear resistance, provided it does not form grain-boundary carbides; (d) usual alloying elements are favorable to wear resistance, as they retard transformation of austenite at subcritical temperatures; and (e) sulphur and phosphorus confer brittleness to steel balls and impair their wear resistance.

Except for some preliminary tests in a small ball mill, results are based on numerous tests run in commercial mills where, along with the regular ball charge, a number of steel balls of known composition and treatment were used. Authors' scrupulous research in this field, where variables are so numerous and difficult to isolate, is a substantial contribution to the scant existing literature. Lelio Stragiotti, Italy

2558. L. Leloup, Tests on thrust bearings (in French), Rev univ. Min. 62, 258–272 (Aug. 1949).

Author describes experimental measurements of friction in three types of thrust bearings under moderate combined axial and radial loads, namely, a deep-grooved ball bearing, a plane-thrust collar bearing, and a thrust collar bearing with beveled sectors. Results are compared with expressions developed by Hanocq for first two cases, while in third case they are compared with hydrodynamic theory for case of a plane slider consisting of an inclined surface followed by a parallel section. Agreement is found to be best at highest loads and lowest speeds, and deviations in reverse case are thought to be due to difficulty of keeping lubricant in resulting large clearance space. Measured friction is compared for the three bearings, and is found to be in decreasing order—plane collar bearing, collar bearing with the beveled surfaces, and ball bearing, assuming that all three are supplied with same oil. Ball bearing will lose much of this advantage if packed with grease. Results show also the advantage of beveling sectors of a plane collar thrust bearing. John T. Burwell, Jr., USA

2559. A. Cameron and Mrs. W. L. Wood, The full journal bearing, Proc. Inst. mech. Engrs. 161, no. 48, 59–72 (1949).

Coordinates of shaft position, the two friction coefficients, and oil flow factor are computed numerically by relaxation method for full journal bearing of finite width having length-diam ratios of 1, $^1/_2$ and $^1/_4$ as well as ∞ on assumption of constant viscosity throughout oil film. Used boundary conditions are that both pressure rise and pressure gradient in direction of motion are zero at trailing edge of the film, and that pressure film always starts at the point of maximum film thickness. For bearing of infinite width, it is shown analytically that these boundary conditions at trailing edge follow from requirement of shaft stability as well as from continuity of oil flow. This effectively takes care of the problem of negative pressures in the oil film. Calculated results are found to agree fairly well with experimental results of Nücker for coordinates of shaft position. They also agree with those of McKee and McKee for friction coefficient, especially as regards showing a positive intercept on friction coefficient axis if shear flow term is taken to extend over 360 deg oil film having bulk oil viscosity. It is also shown analytically for infinitely wide bearing that, as eccentricity ratio approaches unity, the Sommerfeld variable varies linearly with minimum oil-film thickness. Results should be valuable as design data, although reviewer questions generality of assumption that pressure film must always begin at point of maximum film thickness. John T. Burwell, USA

Marine Engineering Problems

(See also Revs. 2284, 2285, 2367, 2477)

2560. E. V. Telfer, Frictional resistance and ship resistance similarity, Shipbuilder 56, 554–559 (Aug. 1949).

One of the major contributors to study of surface friction resistance of ships presents further data involving analysis of resistance of a number of geometrically similar forms as a justification for a different frictional resistance from that predicted by the Prandtl-Schlichting or Schoenherr formulas. A discussion of roughness and of wake and thrust deduction is included.

F. E. Reed, USA

2561. P. C. Lombardini and R. Fidderman, A new theory on propulsion of high speed water craft and its application to the hydroler, Proc. seventh int. Congr. appl. Mech. 2, part II, 533–547 (1948).

Authors give a description with underlying theory of a high-speed planing hull provided with a series of rollers, or drums, rotating on horizontal axes. Rollers are power-driven independent of the main sources of power, and can be lowered when critical stalling speed is approached. When their speed is equal to transitional velocity of craft, it is claimed they will develop high lift at very low drag, thus lifting the planing hull clear of water and allowing much higher speeds than otherwise possible.

Considering the complicated mechanism, vulnerability of the craft in a seaway, and unsolved questions of maneuverability, stability, strength, etc., it is extremely questionable whether these ideas will find practical application. L. Troost, Holland

Biomechanics

2562. N. Rashevsky, Theory of ameboid movements, Bull. math. Biophys. 9, 179–184 (Dec. 1947).

Postulating that the pseudopodia result from a local lowering of surface tension propagated along surface with a constant linear velocity v, author argues that an approximate period $T = P/v$ will result, P being an average meridional perimeter. Following his familiar approximations, he introduces Betti's theorem to obtain expressions for P and hence $T \sim V^{1/3}$. An alternative assumption is proposed, without elaboration, of random location, size, and duration of spots of lowered surface tension.

A. S. Householder, USA